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## MORPHO-LOGICISM AND BEYOND

Morpho-logicism is the theory according to which: (i) reasoning is fundamentally formal in the sense that the validity of an argument depends on its form and not on its contents or meaning, (ii) mathematical reasoning is such a formal reasoning. This doctrine has been promoted and espoused by a variety of people, from Rougier to Curry to MacLane

According to this view, Russell's paradox or Gödel's theorem are not necessarily obstacles to the reduction of mathematics to logic. The non logicallity of notions appearing for example in the axioms of set theory is relativized by the formal aspect of the axioms and the formal deductions derived from them. Axioms of mathematical theories are not considered as necessary truths and in some sense even logical laws and rules are devoid of necessity. Morpho-logicism is compatible with the pluralist approach to logic and it pushes away the distinction between the logical and the non logical.

We will criticize morpho-logicism, showing that it does not really explain what form means and what the relation between logic, mathematics and reality is. We will emphasize the fact that a precise characterization of what logical form is requires mathematical tools, showing that mathematics is needed to develop morpho-logicism. We will also stress the rejection of the paradigm of "logical form" by Wittgenstein, J.-L.Destouches, P.Février and later on by relevant logicians.

Finally we will present some recent works towards a new theory of reasoning based on some experiments on the brain, where logical form appears in a different perspective.

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## CONSTRUCTIVE SET THEORY

Constructive set theory originates in the early seventies from Myhill's endeavors to provide a formal system in which to codify the mathematical practice of Bishop, as exemplified in Bishop's monograph ``Foundations of constructive analysis'', 1967.

Constructive set theory is one of the various formal systems for constructive mathematics which have been put forward at around the same time; it is receiving new attention in recent years, especially in the form of Aczel's systems CZF (Constructive Zermelo Fraenkel). In the talk we shall first of all recall in more detail the origins of constructive set theory and its fundamental motivation. We shall then briefly present some of the essential aspects of the notion of set it describes and its formulation, and conclude, time permitting, with a hint to some recent results in the area.

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### BROUWER'S INTUITIONISTIC UNIVERSE: BETWEEN LAW AND CHOICE

In 1907, in his dissertation, Brouwer presented his first constructive program; it was based on the so-called ur-intuition, which provided the building blocks for enough mathematical objects to handle the basic parts of mathematics. There is little doubt that he grasped the role and the basis of constructive logic, although Brouwer rejected the principle of the excluded middle one year later. The real breakthrough took place in 1916 when Brouwer saw how to handle choice sequences. On the basis of his principle `there are no objects but those created by the human subject', the could only obtain a limited mathematical universe, which nonetheless was extra-ordinarily rich, because of the nature of its objects and of a suitable logic. Brouwer enriched mathematics by his refined view of N and N^N (Baire space), and his `logic through constructions'. After the euphoria of the continuity phenomena, Brouwer spent years to explore the range of consequences of the notions of choice and the creating subject. We will present a survey of the potential and the problems of Brouwer's universe.

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### VARIATIONS OF FREGE'S *GRUNDGESETZE*

There are several causes of the inconsistency of Frege's *GGA* (*Grundgesetze der Arithmetik*). The most prominent is the wicked half of *GGA V* which postulates that the operation (the so-called *spiritus lenis*) which takes functions to their Werthverläufe is injective. However, there are at least two further causes. One of them is an impredicative comprehension (or substitution) rule; another is the fact that the *spiritus lenis* is a total operation.

In my old book [1][1] I have set up a system called *präKid* which is strictly predicative and retains all other dangerous laws of *GGA*. I do not yet know whether *präKid* is consistent. It is a second-order system and derives large parts of Quine's *NF* under at least two embeddings (e.g. the first eight axioms of Hailperin's nine-axiom formulation of *NF*). Nevertheless, *präKid* is a flat system since it seems that

[1]The mathematical stuff of this book (which does not immediately concern Frege's Logicism) has been rew¨orked in [4].

the general powerset axiom cannot be embedded. Of course, I hope that the system is consistent. However, even under that supposition, there are, among others, the following drawbacks.

(1) The notion of predicativity has been formalized within *präKid* in a rather suboptimal manner; even fragments of $\Delta^1_1$-comprehension make the system inconsistent. It follows that any real use of the system will be ridiculous. [Of course, there seems to be no actual use of any variations of *GGA* in prospect.]

(2) I suspect that the consistency strength of *präKid* is very small, somewhat about that of a tiny fragment of arithmetic.

Therefore I have played around with some new modifications of *präKid*:

(A) Enhancement by introducing types. Most important are *cyclic types*. For instance, type 1 is in type 2, type 2 is in type 3, ..., type 99 is in type 100, and type 100 is in type 1. The arising systems are bound to be strictly predicative; but they seem to be richer than the original *präKid* which they include. Furthermore, the idea of cyclic types is yet unexplored although it is just a generalization of Frege's duality of function and Werthverlauf.

(B) Deviating from the possibilities in (A), we may stick to second-order logic and add ramification indices in order to refine predicative comprehension. Here I have proved (nonconstructively) the consistency of a system with ramification indices below the ordinal $\omega^\omega$. This system seems to be much stronger than those with cyclic types.

One third of my talk will be devoted to more philosophical problems about Logicism. First of all, what one could call Frege's fundamental laws of first-order arithmetic can easily be formulated in a logic-free equation calculus. Why is there any special need of logic at all in the "foundation" of arithmetic? Here logic is simply quantification theory. Is there a *different* need of logic in the foundation of geometry? Next, we may speculate how Frege could have proved several substantial number-theoretic theorems within *GGA*, e.g. the infinity of primes (in arithmetical progressions).

I claim that the systems in [3] solve the problems of Russell's Logicism in a satisfactory way since they embed (1) *Principia Mathematica* and are (2) *purely logical* in a precise way. In addition, (3) they satisfy the cut elimination theorem.

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### QUANTIFICATION AND IMPREDICATIVITY

Impredicative functions – understood as functions transgressing the vicious circle principle – are usually considered as problematic in a logicist perspective. The purpose of this talk is to show that a general solution to the admission of such functions inside the logical language asks for a re-examination of the nature of quantifiers. In this view, I will focus on a conception called categorical quantification. This conception is mainly grounded on Lesniewski and Ajdukiewicz's theory of semantic categories, which allows to formalize the relation between syntax and semantics in formal languages. I will also describe how such a view on quantifiers allows to deal with impredicativity in a way similar to the one of F.P. Ramsey.

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### LE LOGICISME, LES NOMBRES ET LES OBJETS

Il existe une tension entre la thèse selon laquelle les nombres sont des objets (le «platonisme» de Frege) et les principes généraux du logicisme. L'exposé exposera cette tension et examinera les voies de sa résolution, à la fois dans les écrits de Frege et dans le programme néo-logiciste contemporain.

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### LOGICISME COMBINATOIRE ET THÉORIE DE LA DÉFINITION

Peut-on n'utiliser que des termes logiques et des procédures logiques de preuve pour reformuler l'ensemble des mathématiques? Et que faut-il entendre par «logique»?

L'argumentaire sera le suivant:

1) Le logicisme ne soutient pas qu'on ne trouve en mathématiques que des détachements, des contrapositions de l'implication, etc. (même informulés). Il est reconstructif et non descriptif. Il ne soutient pas non plus que toutes les procédures qui interviennent *de facto* dans une démonstration mathématique (par exemple barrer certains multiplies dans la suite des entiers pour obtenir la suite des nombres premiers) seraient à valider par une traduction juxtalinéaire donnant des correspondants logiques. Il se propose de transcrire toute opération portant sur des objets et qui permet d'établir sur eux une certaine propriété formulée par un énoncé B, en opérations logiques portant sur un énoncé A et en obtenant le même énoncé B. On ne peut pas lui opposer les activités de la pensée vivante.

2) Le mérite du logicisme n'est pas de corriger des parlogismes, mais de traquer des suppositions implicites fausses (généralement responsables des erreurs en mathématiques), en cherchant à donner une *demonstratio ad oculos*.

3) Le problème n'est pas de reconstruire les mathématiques par une logique qui serait restée naïvement elle-même ni, inversement, de reconstruire une logique affinée comme étant une certaine algèbre (par exemple une structure d'anneau pour le calcul des propositions). Beaucoup de disciplines s'hybrident, et utilement, au cours de leur histoire.

4) Le logicisme pose deux grands sous-problèmes:1°) Peut-on définir (ou plutôt redéfinir) les termes mathématiques en primitifs logiques? 2°) Peut-on démontrer (ou plutôt re-démontrer) les théorèmes mathématiques à partir des seules règles logiques et sans utiliser de prémisses extralogiques? On ne répondra que brièvement à la deuxième question, pour s'intéresser à la première qui est basale.

5) Il est connu que les *foncteurs*, les *quantificateurs* et l'*appartenance* suffisent à «remplacer, comme dit Quine, toutes les notions de l'arithmétique, de l'algèbre, du calcul différentiel et intégral, ainsi que des branches des mathématiques dérivées» qu'ils permettent de définir. Nous ne chercherons pas comment cela s'effectue, mais s'il s'agit de notions logiques et ce qui conduit à le souhaiter.

6) On ne supposera que la conception classique des définitions car la logique combinatoire qui va intervenir exigerait une adaptation antérieure. En dialoguant avec plusieurs auteurs, on défendra l'idée que les primitifs sont choisis pour leur intelligibilité sans que cela conduise à un «psychologisme» indu. Une partie des vues contemporaines sur le logicisme peut d'ailleurs dépendre de la manière dont on a su récemment réévaluer ce qui a été nommé «psychologisme». Le logicisme n'est pas seulement une thèse sur les mathématiques, mais aussi une thèse sur l'intelligibilité apportée par l'usage de notions d'un certain type.

7) Après une présentation à grands traits de la logique combinatoire, on montrera que tous les *foncteurs* et les *quantificateurs* peuvent être reformulés (ou évités) par des combinateurs, conçus comme les notions logiques sans doute les plus élémentaires et les plus directement intelligibles. Ce sera là, au plan technique, la part contributive de l'exposé. On peut la considérer comme l'expression (au moins méthodologique) d'un logicisme combinatoire. On accorde que son radicalisme a notamment pour objectif de provoquer des prises de position sur la reconstruction logique des mathématiques.

8) En réponse à Quine, on défendra l'idée que l'*appartenance* est une notion logique. Son emploi, en effet important et risqué, en théorie des ensembles ne conduit pas à dire qu'elle serait une notion ensembliste ni que la théorie des ensembles, en effet mal assurée, est une logique, ce qui affaiblirait celle-ci sans renforcer les mathématiques.

9) Au demeurant, le logicisme ne se mesure pas aux services qu'il rendrait en mathématiques. Une reprise logiciste – combinatoire ou non, complète ou partielle – peut nous apprendre sur l'intelligible et le rationnel plus que sur les nombres et les figures, et sans démériter.

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### ABSTRACTION ET NOMINALISATION

C'est à l'aune du logicisme construit dans le cadre catégoriel et développemental de l'Ontologie de Lesniewski que s'inscrit cet exposé. Notre réflexion se déroulera autour de la possibilité qu'offre un tel système, sur un plan linguistique, de *parler* les entités abstraites *comme si elles étaient des objets*, sans être contraint par une quelconque implication ontologique. Trois traits essentiels à notre propos et configurant ce paradigme logique sont à relever. Tout d'abord la quantification est ontologiquement neutre. Ensuite, l'analyse de la proposition n'est pas tributaire du modèle mathématique et de la distinction fonction/argument : la proposition élémentaire est de la forme *a est b*, formellement $a \varepsilon b$, où l'epsilon a statut de foncteur primitif et *a* et *b* sont des objets formels appartenant à la catégorie des noms. Dans ce cadre d'analyse, il n'est nullement question de classe –ou d'ensemble- mais simplement d'extension attachée à un nom, celle-ci pouvant être singulière, vide ou plurielle. Enfin le dernier trait est celui de la dimension développementale ou constructive de l'Ontologie, celle-ci étant portée par une procédure inférentielle définitoire qui assure au langage une constructivité catégorielle potentiellement infinie.

Par la possibilité de définir des epsilons d'ordre supérieurs et d'élever l'axiomatique de l'Ontologie aux catégories de ces foncteurs, il est possible de nominaliser les entités supérieures sans qu'il soit nécessaire de procéder à leur réification. Les difficultés liées au processus de nominalisation que rencontra le logicisme classique se trouvent ainsi déliées, puisque la représentation des entités définies ne s'accompagne pas d'une augmentation de l'ontologie première par l'adjonction d'«objets » mathématiques. La pensée logiciste peut dès lors s'inscrire et se développer hors du réalisme dogmatique de la théorie frégéenne ou du nominalisme instrumental des *Principia Mathematica* où, si l'on échappe également à une réification des nombres, c'est au prix de la stratégie de la théorie des fictions logiques visant à sauver le programme universaliste. Conciliant la notion de nom et celle de fonction par un réglage formel du processus linguistique de nominalisation, l'Ontologie présente alors le mérite de nous libérer du scandale de nommer les fonctions et d'en faire les sujets logiques d'assertion, conséquence fâcheuse de la doctrine des symboles fonctionnels comme écritures incomplètes.

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## KIT FINE ON *THE LIMITS OF ABSTRACTION*

Although Kit Fine allows that a significant theory of abstraction can be developed in which some fundamental mathematical theories may be reconstructed, the overriding message of his book, as its title suggests, is largely negative. In particular, Fine is quite out of sympathy with the idea that abstraction might play any significant or useful rôle in providing a philosophical foundation for mathematics—even for those parts of which can be obtained on abstractionist principles. I suspect that, given certain assumptions underlying and shaping Fine's approach to the subject, this negative assessment is more or less inevitable. In this paper, I explore what I believe to be some of the more important of those assumptions, and suggest some reasons why we might not feel under irresistable pressure to subscribe to them.

Abstraction principles are principles of the shape: $\forall\alpha\forall\beta(\Sigma(\alpha) = \Sigma(\beta) \leftrightarrow R(\alpha,\beta))$—where R is an equivalence relation on entities of the type of entities over which $\alpha$ and $\beta$ vary, and, if the abstraction is good, $\Sigma$ is a function from entities of that type into objects. A very well-known and much discussed example—which figures centrally in Fine's critical discussion—is Hume's principle, which is a second-order abstraction (or a conceptual as opposed to objectual abstraction, in Fine's terms) over an equivalence relation on concepts, asserting that the cardinal number of a concept F is identical to the cardinal number of a concept G if and only if there is a one-one correspondence between the Fs and the Gs. In the more philosophical half of his book—Parts 1 and 2 of *The Limits of Abstraction*, Oxford: Clarendon Press 2002—Fine discusses whether such principles can serve as a philosophical foundation for any significant part of mathematics. He assumes—and the neo-Fregean abstractionist will agree—that they can do so only if they can function as a acceptable definitions of fundamental mathematical concepts, such as those of cardinal and real numbers. Where Fine and the neo-Fregeans principally disagree is over whether they can do so. Fine presents several arguments for the negative view that they cannot do so. He distinguishes between what he calls classical or orthodox definitions and non-classical or unorthodox ones. As far as orthodox definitions are concerned, Fine argues that they presuppose a domain of discourse—typically a domain of objects to include the intended referents of the defined terms—in a way that prevents the definition from serving in a foundational capacity, because the presupposition requires independent justification. He infers that the only way in which one might hope to base any mathematical theory on abstraction principles is by holding that they can function as definitions of an unorthodox or non-classical variety. This, he thinks, involves treating them as creative definitions. Under the heading of unorthodox definitions, Fine mainly discusses what he calls definition by reconceptualisation and implicit definition. He argues that neither kind of definition can work—more precisely, he allows that there are plenty of good implicit definitions, such as the standard recursive definition of addition in terms of the arithemetical primitives natural number, zero and successor, and that there may be successful examples of definition by recomnceptualisation. But he thinks that abstraction principles like Hume's principle cannot function as definitions of either kind.

Some of Fine's arguments for his negative conclusions about the possibility of using abstraction principles as definitions appear to me to depend upon a general assumption or assumptions about definitions which is independently questionable and which the neo-Fregean can certainly reject. Others of his arguments rely upon further assumptions—specifically about the notion of reconceptualisation (which has played a key role in neo-Fregean discussions) and about the way in which implicit definitions work—which the neo-Fregean is also free to reject. In my talk, I will try to explain and justify these claims. I will also, if I have time, try to say a little about the conception of abstraction underlying Fine's approach to the issues that divide us, and explain why I do not think we have to adopt it. Very broadly speaking, I think Fine adopts what could be called an *externalist* standpoint with regard to abstraction. Centrally, this involves thinking that what objects there are is—if not completely, the at least to a very large extent—independent of abstraction. The acceptability of abstraction principles is then to be viewed as a matter of their being true of a certain domain—the universe of all objects whatever—of independently constituted composition and cardinality. Under a suitable assumption about the size of this all-inclusive domain of objects—that the cardinality of the domain is unsurpassable—the acceptable abstraction principles can be characterised as those which are non-inflationary and predominantly logical. But abstraction principles themselves cannot be used to give us any assurance of the truth of such an assumption, which must be justified independently, if it can be justified at all. This is why, in Fine's view, abstraction principles cannot provide a philosophical foundation for mathematics; and for essentially the same reason, they cannot furnish any free-standing, independently viable means of defining concepts or objects. However, whilst this pessimistic assessment of the abstractionist's prospects may be difficult, if not impossible, to resist, if one assumes the correctness of an externalist standpoint, it is very much open to question whether one must—or even can—adopt that standpoint. Not only does Fine fail to provide any compelling reason to accept it—there are, on the opposite side, serious reasons to doubt its intelligibility.

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## ON SOME CONSEQUENCES OF THE DEFINITIONAL OPACITY OF HUME'S PRINCIPLE

Neologicism is the thesis that arithmetical truths are analytic, them being logical consequences of an analytic truth, namely Hume's Principle. The neologicist seeks to clarify the epistemological value of this result by telling the story of Hero, a subject who masters second-order logic and comes to acquire knowledge of arithmetical truths simply by reflecting upon the logical consequences of HP. One of the objections to the analyticity of HP, the so-called *bad company objection*, states that Hume's Principle cannot be thought of as analytic since it belongs to a genre of principles, *abstraction principles*, which has inconsistent instances. Wright's strategy for dealing with this objection is to introduce some restrictions (e.g. consistency, conservativeness, modesty) on the acceptability of abstraction principles. The introduction of restrictions forces the neologicist to endorse the following thesis:

> (**Definitional Opacity**) For most abstraction principles AP, Hero cannot prove that AP is good.

This is shown by some results in mathematical logic, such as Gödel's second incompleteness theorem. Since Hero's story is intended to show the epistemological innocence of Hume's Principle and of its consequences, endorsement of **DO** amounts to the fact that, when inquiring after the epistemological status of HP, we cannot presuppose a demonstrative warrant for its truth.

I explore, then, what happens when one accepts **DO**. Three points are more salient: (1) The neologicist claims that Hero, by stipulating HP, establishes its truth and comes to know it. But can we really say that Hero *knows* HP? In fact there are many inconsistent principles which Hero could have stipulated and which she has no means to distinguish from HP. I argue that the situation resembles the barn-example in the theory of knowledge. (2) The neologicist is committed to the thesis that a bad definition is one that fails to introduce a concept. But **DO** opens up the possibility of a scenario where Hero, while building his a priori knowledge, stipulates a bad definition. It seems that, having accepted a bad definition, Hero can go on using it. Hence, the neologicist has to give an account of Hero's apparent use and explain why Hero's use does not count as a *proper* use. (3) Given **DO**, it does not seem right do describe Hero's attitude as that of *establishing* the truth of HP. Rather, he seems rightly described as merely *accepting* the principle *as* true.

I conclude my paper by offering an alternative account, according to which a stipulation is only an *attempt* to capture a truth. I suggest that the moral we should draw is that the truth of a stipulated principle is never established by us, even when the principle *is* true. I also show that this picture better mirrors and explains our actual practices of introducing, and sometimes modifying or even rejecting concepts.

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### A "CONSTRUCTIVE" VIEW ON LOGICISM

Since its very beginning in Frege's work and then in Whitehead and Russell's *Principia Mathematica*, logicism has been widely thought to be a *reductionist* program. This can be understood on different levels. One of the basic logicist claims is that mathematical notions can be reduced by definition to logical ones and also that mathematical proper axioms and theorems are eventually theorems of logic. On the other hand, it has also been supported, on a epistemological point of view, that mathematical truths can be justifed on a purely logical ground. As we know, the strong reductionist claim that mathematics (in particular arithmetic) is nothing but logic has been historically considered as a failure. Frege's program led to inconsistancy and the authors of the *Principa* had to enlarge their axiomatic basis, including three non logical axioms. Faced with this failure, neo-logicist programs have certainly to propose a way to undertsand the reduction process in a sense which does not lead to the strong (and probably excessive) view that mathematics is nothing but logic.

It is a matter of fact that a central difference between the logicist programs nowadays available is due to the different uses and conceptions of definition. Even if it is still accepted by numbers of logicians as a standard view, Russell's conception of definition as a procedure of sheer linguistic abbreviation, external to the official logical tools, is clearly too poor and limited for the purpose of logicism. Differently from the neo-Fregean School of St Andrews (led by B. Hale and C. Wright), the logicist program intiated in Neuchâtel does not break these limitations in allowing the use of *implicit* definitions such as Fregean abstraction principles. Following and developing a conception of definition initiated in the Varsaw School of logic (especially by S. Lesniewski, A. Tarski and C. Lejewski), I will show that *explicit* definitions can be much stronger than abbreviative Russellian ones and can be used in several situations as valuable alternatives to implicit definitions.

After a short presentation in this talk of the internal conception of definition used in the program developed in Neuchâtel, I will emphasize on two main points. First, I will show how the very notion of logical system is modified by the adoption of this internal conception. Contrary to the usual view, definitions are considered here as official theses of the system, introducing not only convenient abbreviations but new possibilities of expression. This gives rise to a kind of formal language and system which is not closed by an initial set of formulas and a determined set of theorems. As a second issue, I will discuss the epistemological consequences of the use of such kind of logical formalism as a ground for giving a logicist picture of arithmetic. Arithmetic in this perspective is not properly *reduced* to logic, but it is shown to be consistantly *constructible* from a system of pure logic by the way of a logically controled list of explicit definitions.

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### A NEW LOOK AT WITTGENSTEIN ON SURVEYABILITY OF PROOFS

In this paper, I wish to examine the implications of Wittgenstein's remarks on the surveyability of proofs (mainly in part III of [6]), from a standpoint akin to [2] and chapter 8 of [3]. In *Principia Mathematica* [5], Whitehead and Russell provided what amounts to an interpretation of Peano Arithmetic in a logical system (through explicit definitions of the fundamental concepts in logical terms and a logical derivations of arithmetical theorems on the basis of the logical axioms), which purported to provide it with a determinate meaning and explain its applicability (see, e.g., [4, p. 10]). The idea that logical interpretation of would provide a link between a given axiomatic system and its application is indeed a constant within the logicist "school", from Frege to Carnap. Against this, Wittgenstein argued against this in *Remarks on the Foundations of Mathematics*, that logic could not provide this basis because the logical equivalent of an arithmetical equation such as 200+200=400 is unsurveyable and one must in the end use arithmetic to figure out that it shows that 200+200=400. In this paper, I shall reconstruct Wittgenstein's argument and show its implications for the logicist claim to provide "as it were the attachment" to Peano Arithmetic "by means of which it is plugged in to its application" ([6], § 4). I shall also look at the underlying conceptions of proofs and show that Wittgenstein's argument is based on a notion of mathematical proof which differs from the "Platonist" conception (harking back to Bolzano) adopted by logicists. I shall conclude by drawing some of consequences in line with the standpoint set forth by Crispin Wright in [7] and Jacques Dubucs in [1].

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### A TYPE-THEORETICAL ANALYSIS OF ZERMELO'S AXIOM OF CHOICE

An analysis of Zermelo's axiom of choice in constructive type theory reveals that the problem with it is not the existence of the choice function but the extensionality of it, which is not visible in an extensional framework where all functions are by definition extensional.

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### LOGIC AND ARITHMETIC

Since there are non-sortal predicates Frege's attempt to derive Arithmetic from Logic stumbles at its very first step. There are properties without a number, so the contingency of that condition shows Frege's definition of zero is not obtainable from Logic. The point has consequences for Hume's Principle, and for separating numbers from physical objects like Julius Caesar.

Refined discriminations between referring phrases to concepts and open predicates enable us to oppose Frege's thought that concepts are categorically distinct from objects. When we are talking about concepts we are merely talking about certain objects by nominalising the associated predicates. And just that gives us a way of separating out concepts from physical objects like Julius Caesar, while allowing them both to be objects. Frege believed that numbers were objects, even though he could formulate no clear way of separating them from other objects. But it was Frege's identification of numbers with objects rather than concepts which supported the specific reasoning which led him to separate objects from concepts. So we inspect his reasons for that identification. What Frege did not fully appreciate was that corresponding to the referential and descriptive uses of numerals, there are complete and incomplete expressions with all predicates. Thus the referential phrase 'the number 7' is complete, but in the incomplete predicate '(7x) x' the same numeral is adjectival, although both relate to that property of discrete and distinctive things of having a correlation with the non-zero numerals up to 'seven'.

The condition about discreteness is crucially involved in the proof that Julius Caesar is not a number. The main consideration which was involved in Frege's judgement that numbers were objects was the contrast between, for instance, 'the number of planets is 9' (Nx:Px = 9) and 'there are exactly 9 planets' ((9x)Px). Only the former represents '9' as a singular referential term, and so Frege took it to be the basis of his formal analysis of Arithmetic. But it is the predicative form which has priority, and it is that fact which also shows that numbers, while still objects, are categorically distinct from objects like Julius Caesar, since they then cannot be known independently of predications. By contrast, one does not need to know someone is Julius Caesar before one can be acquainted with him. The priority of the predicative form arises because from '(nx)Px' there follows quite straightforwardly 'εm(mx)Px = n' (i.e. 'Nx:Px = n'). But the reverse entailment crucially does not hold, because of the numerical indeterminacy of non-sortals.

That fact also undermines entirely all Fregean, and Neo-Fregean attempts to derive Arithmetic from Logic. Boolos and Wright, with others, have demonstrated how most of Frege's development of Arithmetic can be obtained from Hume's Principle, starting from Frege's definition of zero as the number of things which are not self-identical (Nx:x≠x). But in this extensive, and now very elaborate discussion, no question has been raised about whether '¬(∃x)(x≠x)', entails '(0x)x≠x', and so 'εm(mx)x≠x = 0', i.e. 'Nx:x≠x = 0'. If the negative existence statement entails the numerical statements, then Nx:x≠x must be determinate, and that is contingent on 'x≠x' being a sortal predicate. Much ink has been spilled debating whether Hume's Principle is analytic, and so whether the Arithmetic taken to be derivable from it can, or cannot, be properly described as a part of Logic. But if Logic does not discriminate between sortal and non-sortal concepts, then there is no way to get from it the other crucial element in Frege's generation of the number series: its starting point. The possibility of non-sortals also reveals that Hume's Principle is false. For two null predicates, whose instances can, therefore, be put vacuously into one-one correspondence, do not necessarily have the same number, because they may each be numberless.

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### MATHEMATISCHES SEMINAR, JENA: SOMMERSEMESTER 1889. ON THE PURELY TECHNICAL ORIGIN OF FREGE'S DISTINCTION BETWEEN *SINN* AND *BEDEUTUNG*.

In the SS of 1889 Frege studied Dedekind's then recent *Was sind und was sollen die Zahlen?* [1888] with two students. The ensuing 1889 revisions in the *Begriffschrift* are prompted by his attempts to formalize Dedekind. The proof of Dedekind's recursion theorem makes use of ordered pairs. Frege's,on the other hand, treats of ordered pairs as iterated *Werthverläufe,*which requires categorical homogeneity between what sentences *bedeuten* ["stand for"] and *Werthverläufe*. From this the idea of truth-values *as objects* is forced upon us, and with that such a distinction as that between *Sinn* and Bedeutung.

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### CONSTRUCTIVE FINITENESS

Dirichlet's Schubfachprinzip, aka Pigeonhole Principle, states that if k pigeons are put into l pigeonholes, and if k > l, then at least one pigeonhole must contain more than one pigeon. In a set-theoretic framework this principle reads as "Every injection on a finite set into itself is surjective." and can easily be shown in a classical setting

In Constructive Set Theory some more care is needed also because there are at least 4 different notions of finiteness to distinguish, e.g. finite and finitely enumerable. They are classically equivalent.

I will present the different notions of finiteness and, if time permits, a proof of the Pigeonhole Principle for finitely enumerable sets.

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## AJDUKIEWICZ AND KOTARBINSKI ON NAMES: A PRETEXT FOR ONTOLOGICAL GAMES

In this presentation I will expose two original ways of thinking to names present in Polish pre-war philosophy, Kotarbinski's reism and Ajdukiewicz criticism of that vision. In order to introduce Ajdukiewicz's conception of names, I will begin by presenting shortly his theory of syntactic connection (*syntaktische Konnexität*) – mainly known from his article published in German, in 1935, in a Polish review *Studia Philosophica*. At the same time, Kotarbinski's conception of names can be understood only in the light of his ontological theory known as "reism" or "concretism". This latter theory, even if originated by Kotarbinski, received a clear and coherent presentation only when it accepted the formal basis of Lesniewski's logic.

It seems obvious, at least in texts of the philosophers quoted above, that the question of names was hiding much deeper quarrels. Although Kotarbinski's and Ajdukiewicz positions were not in radical opposition, several disagreements between both philosophers were very fructuous to their respective works. I would like to try to give an overview of their respective contributions to logic presented against the background of their philosophical positions.

It would not be easy to talk about Ajdukiewicz's logic without mentioning his epistemological point of view, known as "radical conventionalism". As he had to defend his opinions while accused of idealism, he was compelled to elaborate an elegant defence of a certain conception of language. And although the philosopher does not seem to be interested in the origins of our language, he provides an interesting theory of how it functions – at this point, one cannot evade from some parallels with Wittgenstein. Even if Ajdukiewicz explicitly rejected some of his radical opinions, he still preserved a very minimalist version of his conventionalism. And the explanation of this reduced theory will enable us to understand the general perspective of his philosophical choices.

As for Kotarbinski, his theory of names as exposed in *Elementy* (or, according to English translation, in *Gnosiology*) and as preserved in his posterior papers gets a number of elucidations coming from reading it in the light of a wider context of other works of philosophers influenced by Lesniewski. His radical nominalism and materialistic monism as ontological postulates remained in deep conflict with Ajdukiewicz's whole methodological approach. The latter was relentlessly reducing the field of application of his ideas, whereas his results seemed to contain some definitive solutions to some semantic problems (what "semantic" meant in Poland in thirties is another riddle). It seems that the most perturbing element in this conflict is the problem of limits of what can be said and of limits of logic as *organon*.