Crucial weights

Towards an understanding of weightings by partial order theory

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Bruggemann_uncertainty_Neuchatel.pptx

The basic problem

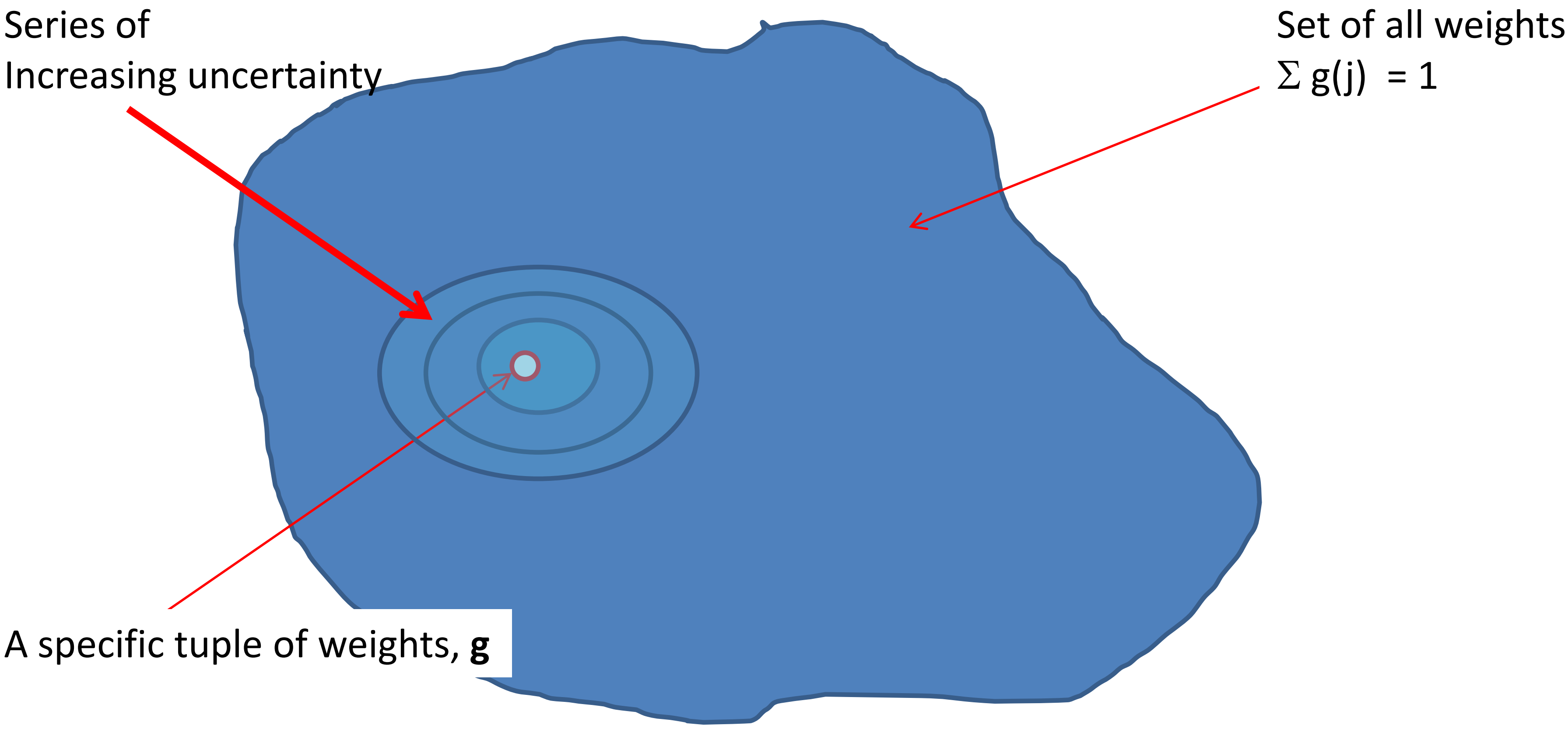
- Posets support decisions?
- Why yes?
- Why no?

Notations and basics

- $(X,\leq)$ the poset based on m indicators and X the set of objects;
  Objects denoted as x(i1), x(i2) or just by i1,i2
  Entries of the data matrix: x(i,j) for the ith object and the jth indicator
- g(j) the weight for the jth indicator
- DSS-Model:

$$CI(i,\mathbf{g}) = \Sigma\, g(j)*x(i,j);\quad x(i,j) \in [0,1],$$
$$\mathbf{g} = (g(1), g(2),\ldots,g(j),\ldots,g(m)) \in G$$

The technical problem is twofold:



(1) Select a tuple **g**
(2) Let us model the uncertainty by (mathematical) environments around **g**

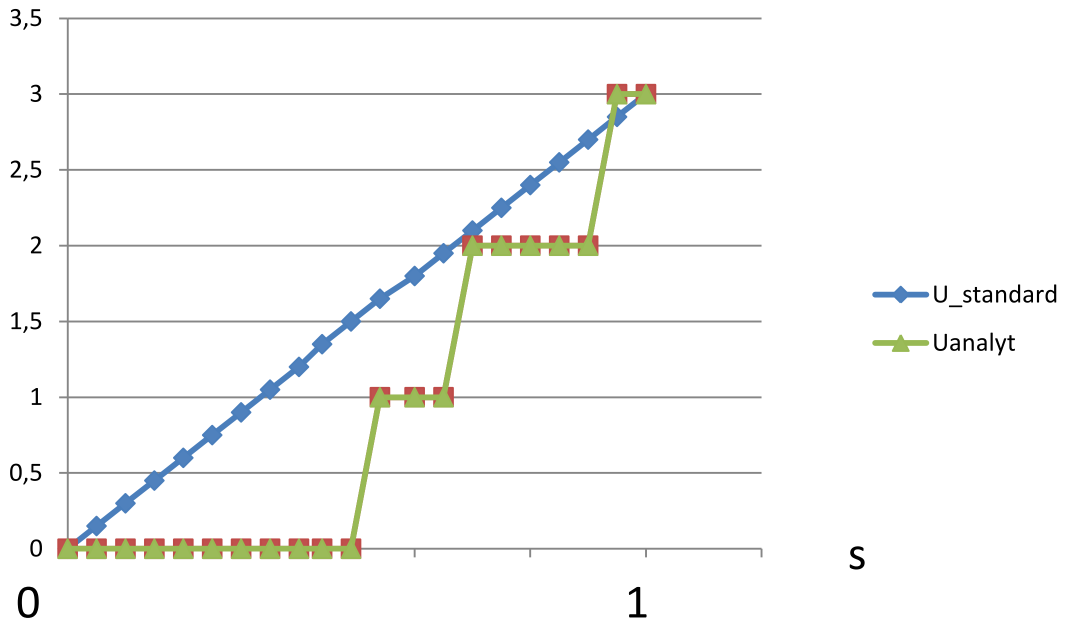
Need of a control function

- Selection of a starting tuple **g**
- System of environments around **g**: Env1(**g**) $\subseteq$ Env2(**g**),…
- A control function is needed to check the effects of uncertainty with respect to the weights selection.
- This control function is $U = |\{(x,y) \in X^2 : x||y\}|$

U = s*U0 is a good approximation [Bruggemann, Carlsen, 2017]…

…with s $\in$[0,1]a measure for the uncertainty in weights
…and U0 being the number of incomparabilities if s=1
(i.e. all weights possible, the original poset
based on the indicators)

Fictitious example: 14 objects, m = 3 indicators



Cont‘d

Understanding of the deviations from the line U=s*U0, the „fine-structure“.

Analysis of the simplest system for posets with some incomparabilities

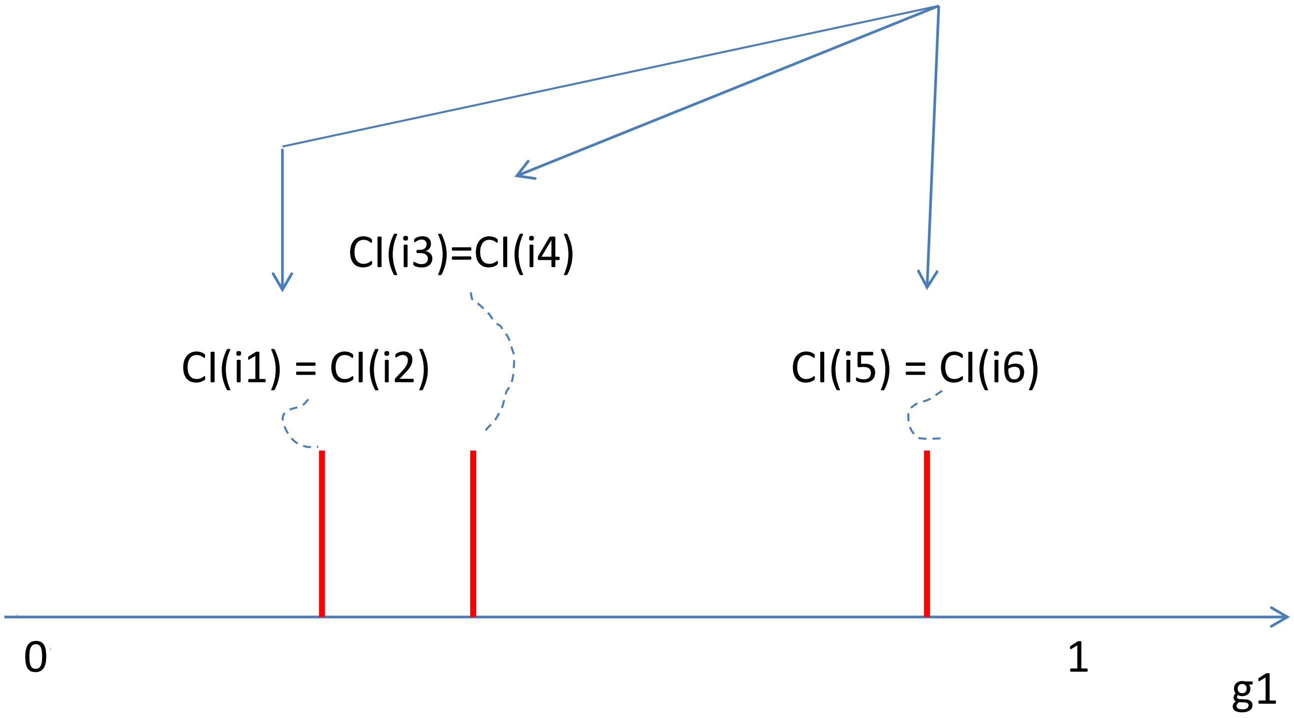
an m=2-system

Motivation: Crucial weights

- Consider two objects A and B and two indicators:
- A = (0.3, 0.6) and B = (0.4, 0.5): A||B
- Weights g1, g2 with g1+g2 = 1
- CI(g1=0.3,g2=0.7; A) = 0.51; CI(...;B) = 0.47:
  $\Rightarrow$A > B
- CI(g1=0.7,g2=0.3;A) = 0.39; CI(...;B) = 0.43
  $\Rightarrow$ A < B
- Crucial weights defined by the requirement:
  CI(A) = CI(B)

Crucial weights

Three crucial weights for six objects (i1,..,i6),
assuming three Incomparabilities (i1,i2); (i3,i4); (i5,i6)



Two indicators, hence weights g1, g2 (g1+g2 = 1), simplified notation

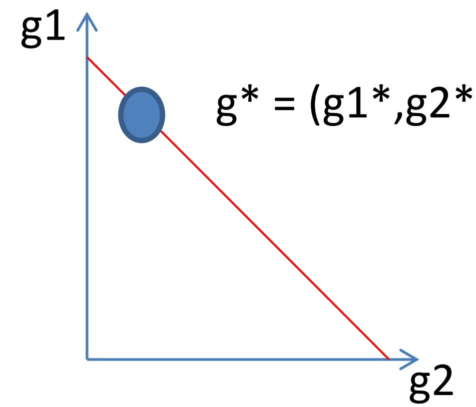
Assume a certain fixed weight g1*:

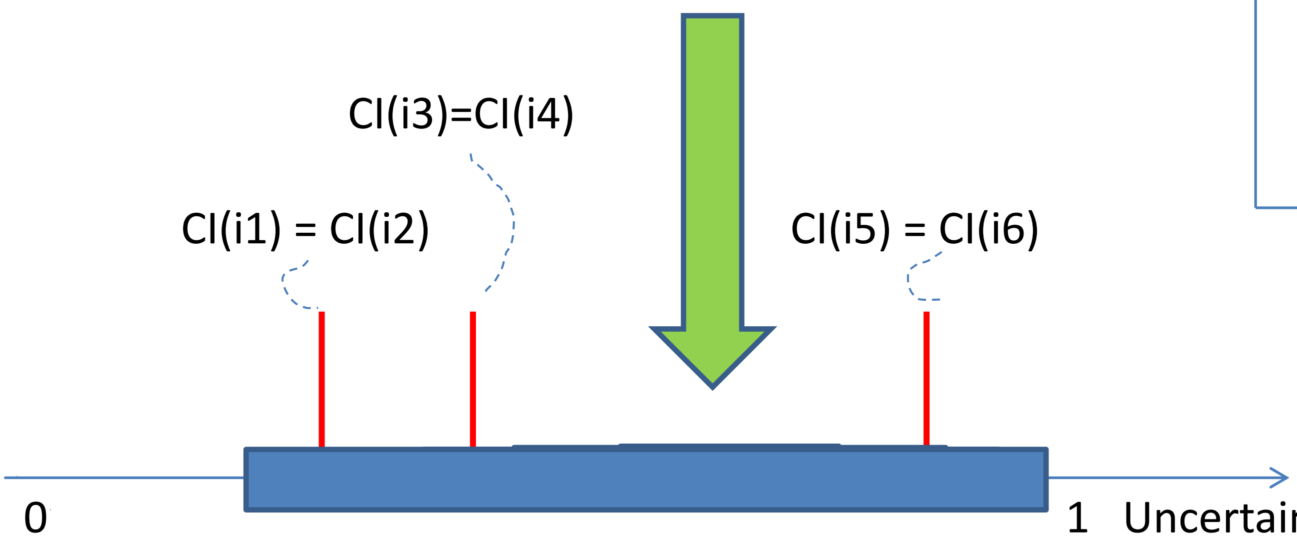
CI(g1*) may lead to:

i1 < i2

i3 > i4

i5 < i6





Uncertainty w.r.t. weight g1

Incomparability increases (starting from g = g*, s=0) with increasing uncertainty interval, when a „crucial weight“ (red vertical lines) is included in the g_{min}—g_{max} interval

For m=2 all the (gc: crucial weight) gc-values can be calculated by a closed formula (Bruggemann et al, 2008)

The distribution of the gc-values is responsible for the deviations from U = s*U0

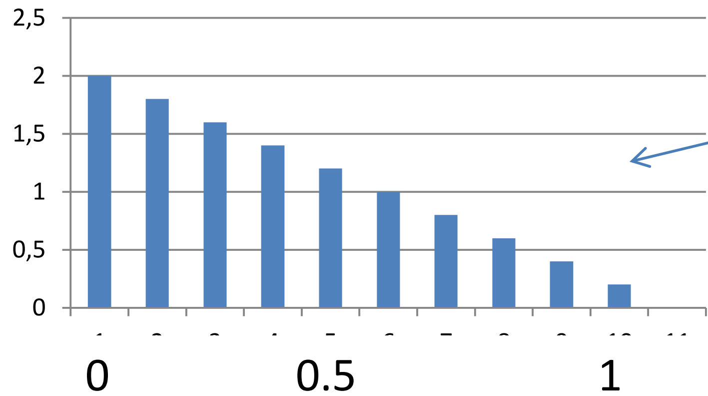
$$U = N * \int h(g,gc)*d\,g$$

- N a normalization factor
- h the distribution of the gc-values, seen as quasi continuous function of g
- Integral from 0 to 1 (!!!)

Examples….

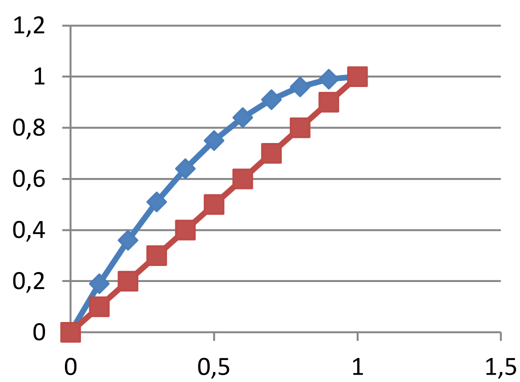
For example: Modelling by $\int h(x)dx$

Idealized: $h(x) = N*(1-x)$, x instead of gc(1) as convenient abbreviation. $x \in [0,1]$



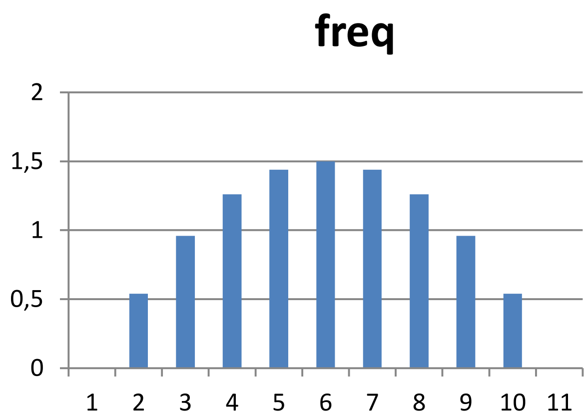
Fictitious example

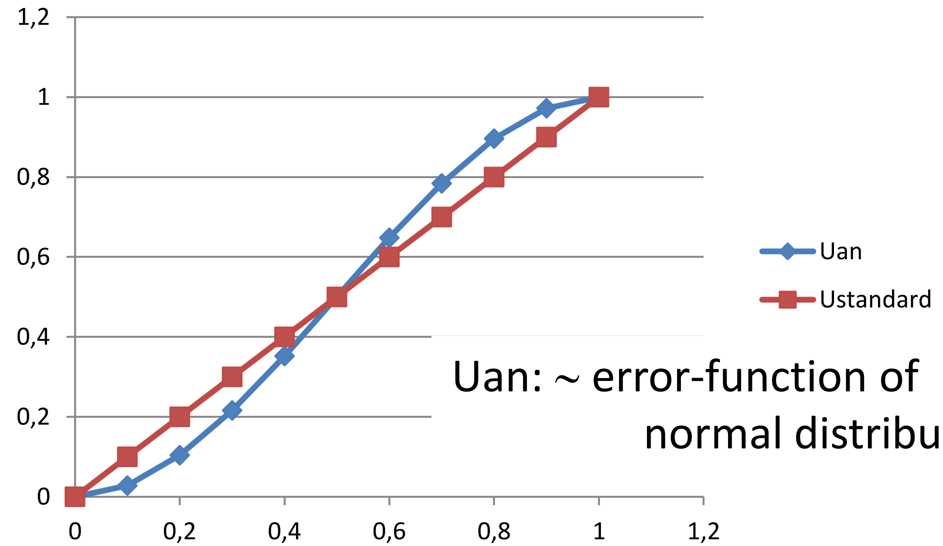
$U = 2*\int(1-x)dx = 2*x - x^2$, U0 for the sake of simplicity = 1



Ustandard = s*U0

The crucial type





Uan: ~ error-function of normal distribution.

This is the most crucial type for weighting processes, because h(x) has its maximum at 0.5 (Nardo Range)

Example: bridge stability

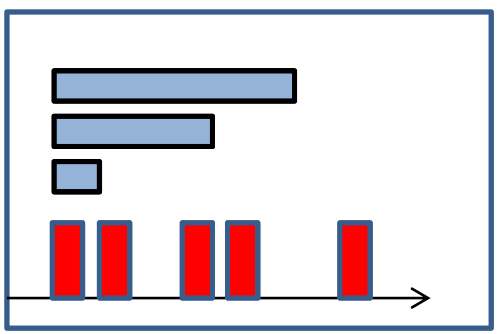
Interim summary

- The deviations from the straight line U = s*U0 are a consequence of the distribution of the crucial weights
- The two dimensional system may be a sufficiently good approximation for a more general system (ambiguiety graph)
- Up to now: start value for g1: 0 , s increases from 0 to 1.

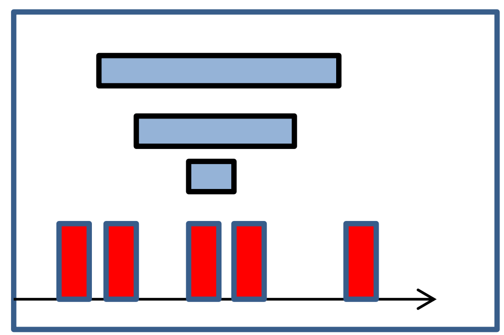
However!!!

- Following Nardo‘s recommendation:
- $g(j) \approx 1/m$, i.e. in an m=2-system. $g1 \approx 0.5$
- We have to take care for the starting tuple **g**
- Starting with weights near 0 is not the standard!!

Not:

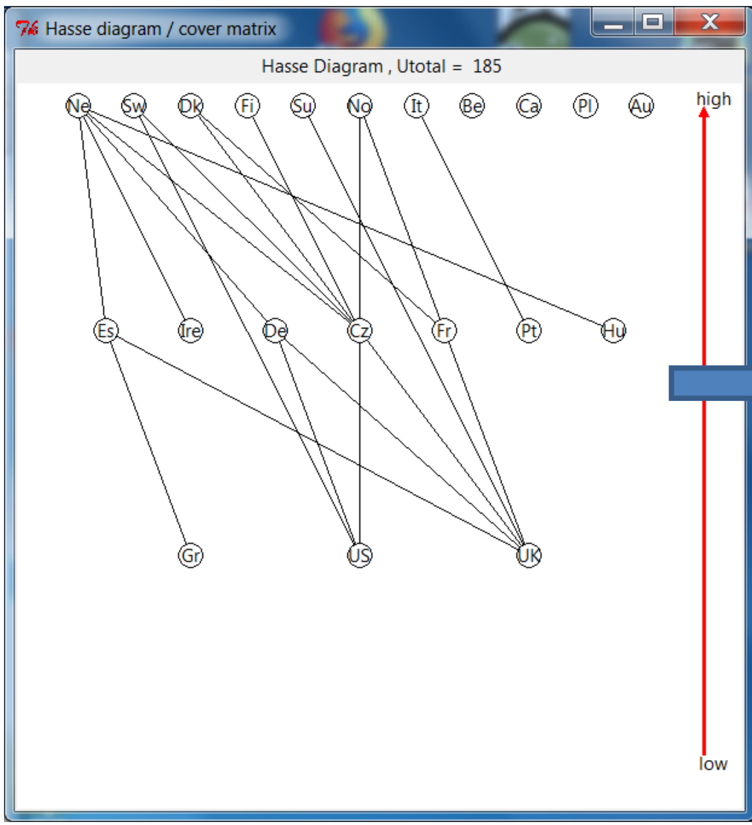


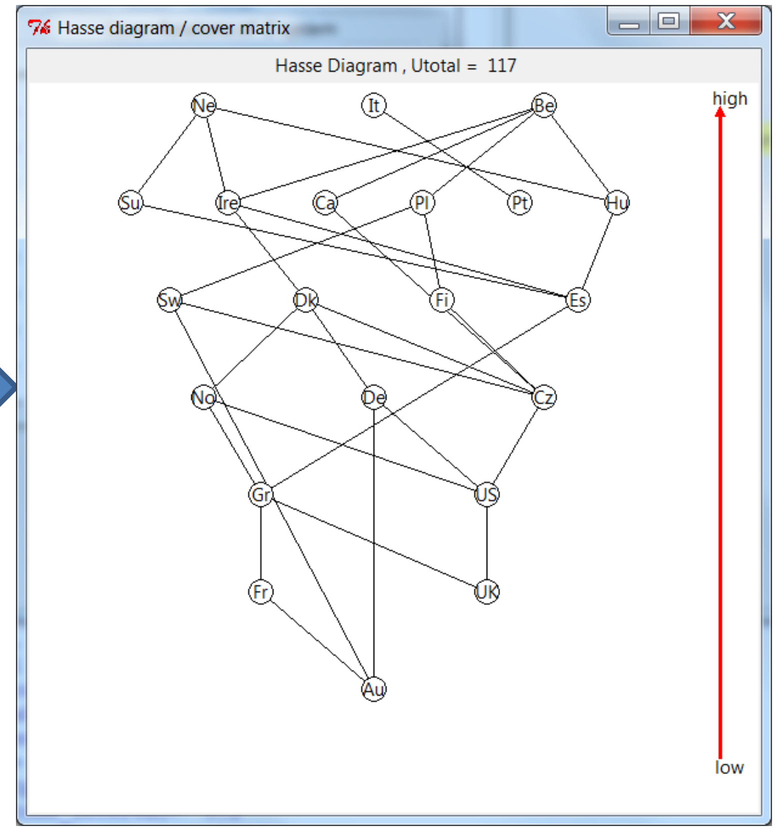
But:



Crucial weights

Child well being

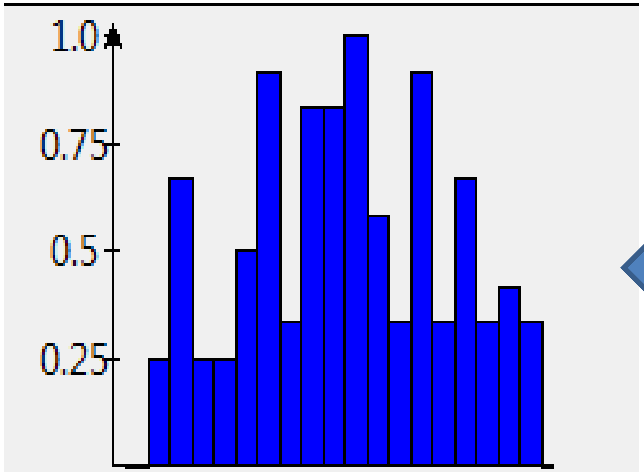




Six indicators: wb, hs, fa, ed, br, sub → 2-indicator system: fa, ed

gc(1): Child well being

Indicators:
wb, hs,
ed, fa,
br, sub,



Actual indicators:
ed, fa

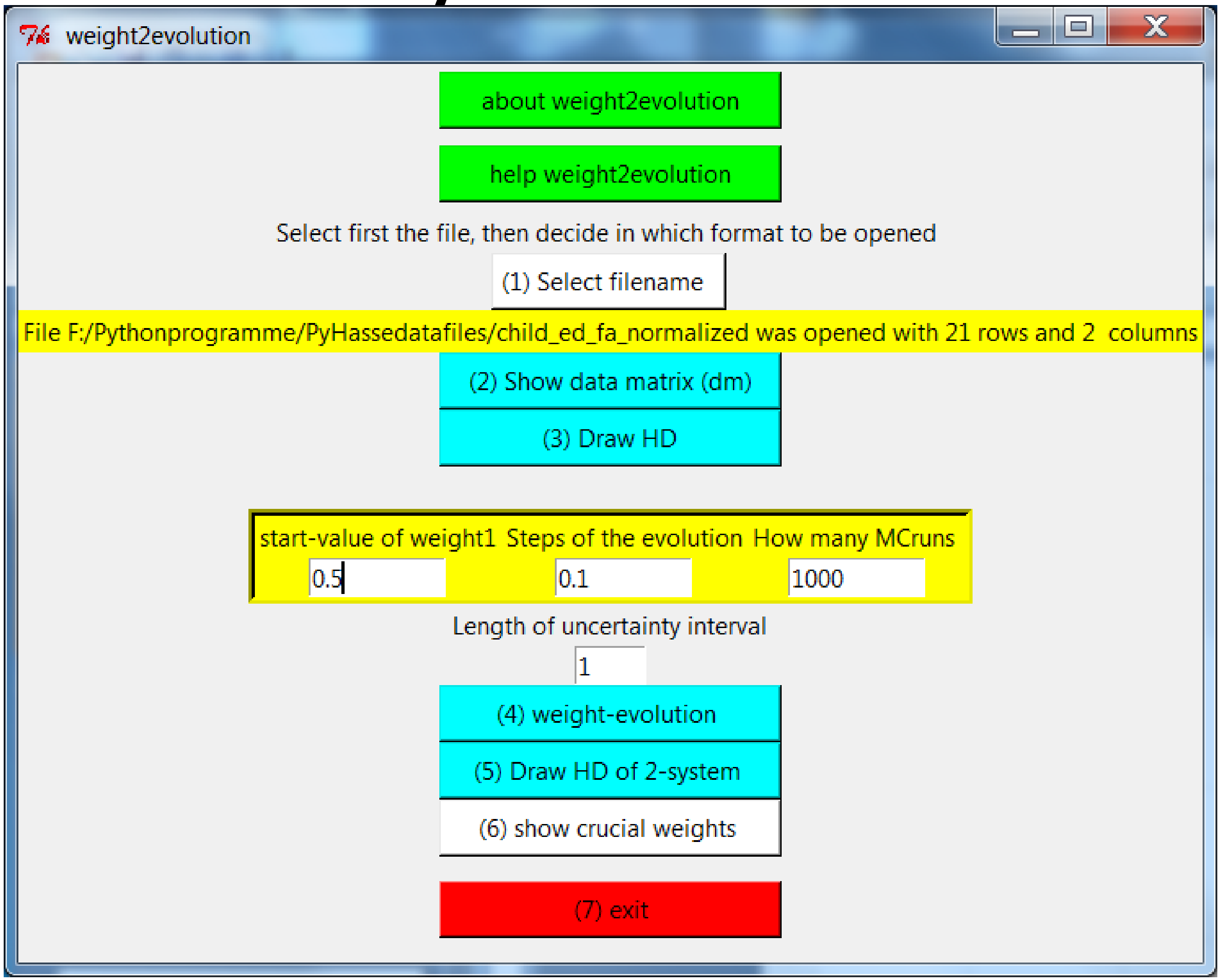
$g1 \approx 0$
Increasing s:
•Few… then
•many, finally
•few
crucial weights

Stable decision situation
i.e. only relatively few incomp.

$g1 \approx 0.5$
Increasing s:
•many then
•few
crucial weights

Unstable decision situation
i.e. relatively many incomp.

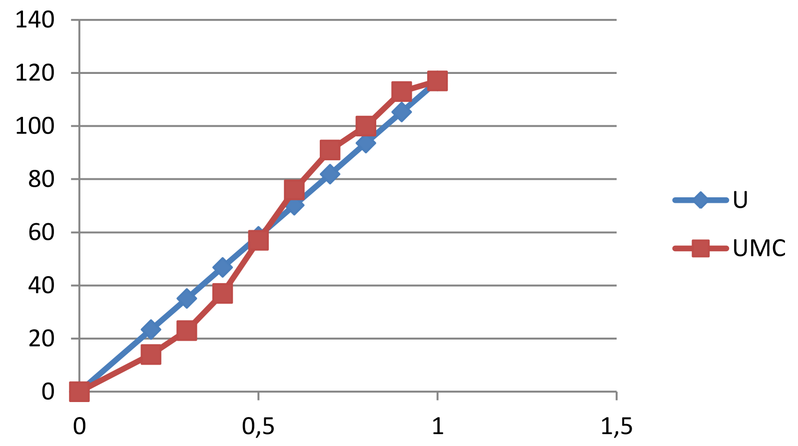
PyHasse



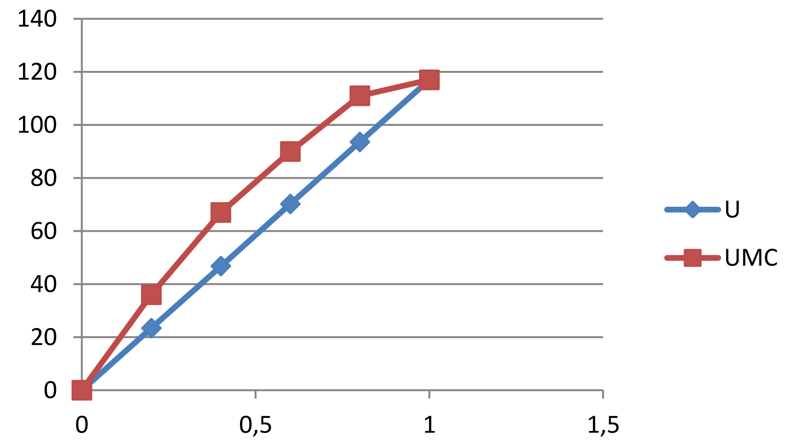
weight2evolution
about weight2evolution
help weight2evolution
Select first the file, then decide in which format to be opened
(1) Select filename
File F:/Pythonprogramme/PyHassedatafiles/child_ed_fa_normalized was opened with 21 rows and 2 columns
(2) Show data matrix (dm)
(3) Draw HD
start-value of weight1
Steps of the evolution
How many MCruns
0.5
0.1
1000
Length of uncertainty interval
1
(4) weight-evolution
(5) Draw HD of 2-system
(6) show crucial weights
(7) exit

Results

a) Start weight: 0.1, s = 0 until s = 1:



b) Start weight: 0.5, s = 0 until s = 1:



Thank you for your attention

Questions,
Comments?

References

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- Nardo, M. 2008. Handbook on Constructing Composite Indicators - Methodology and User Guide. OECD, Ispra.

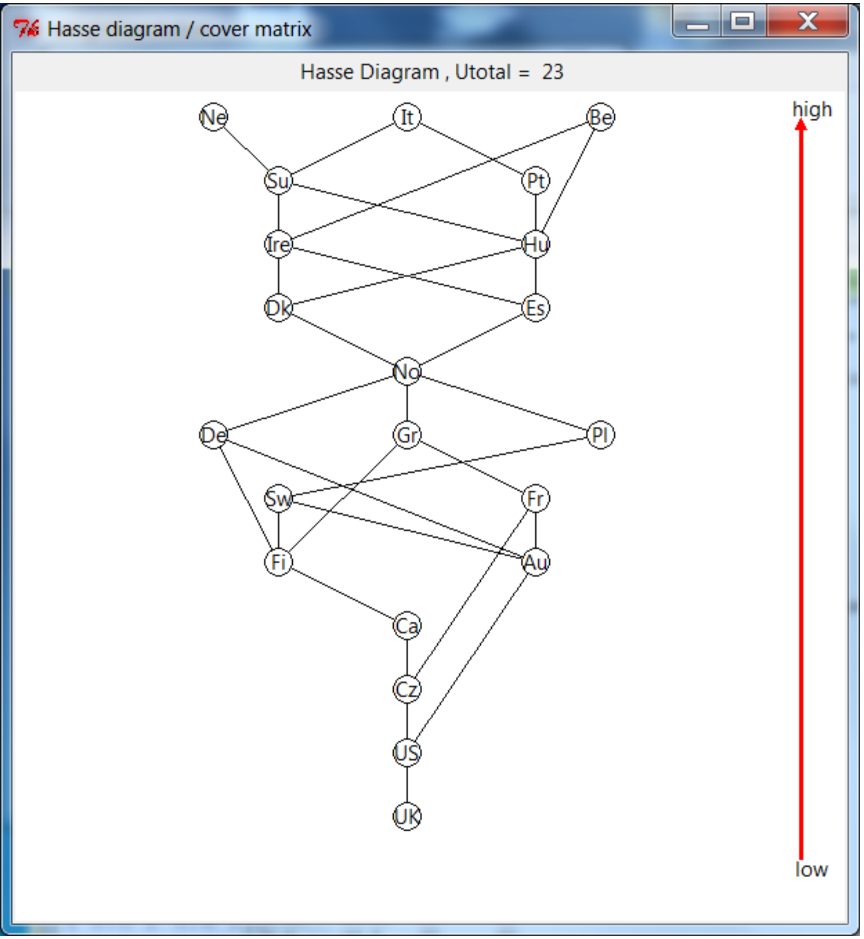
PyHasse, normalization

	ned	nfa
Ne	0.75	0.9
Sw	0.8	0.3
Dk	0.65	0.6
Fi	0.85	0.2
Es	0.3	0.65
Su	0.35	0.85
No	0.5	0.55
It	0.05	1.0
Ire	0.7	0.7
Be	1.0	0.8
De	0.55	0.4
Ca	0.95	0.15

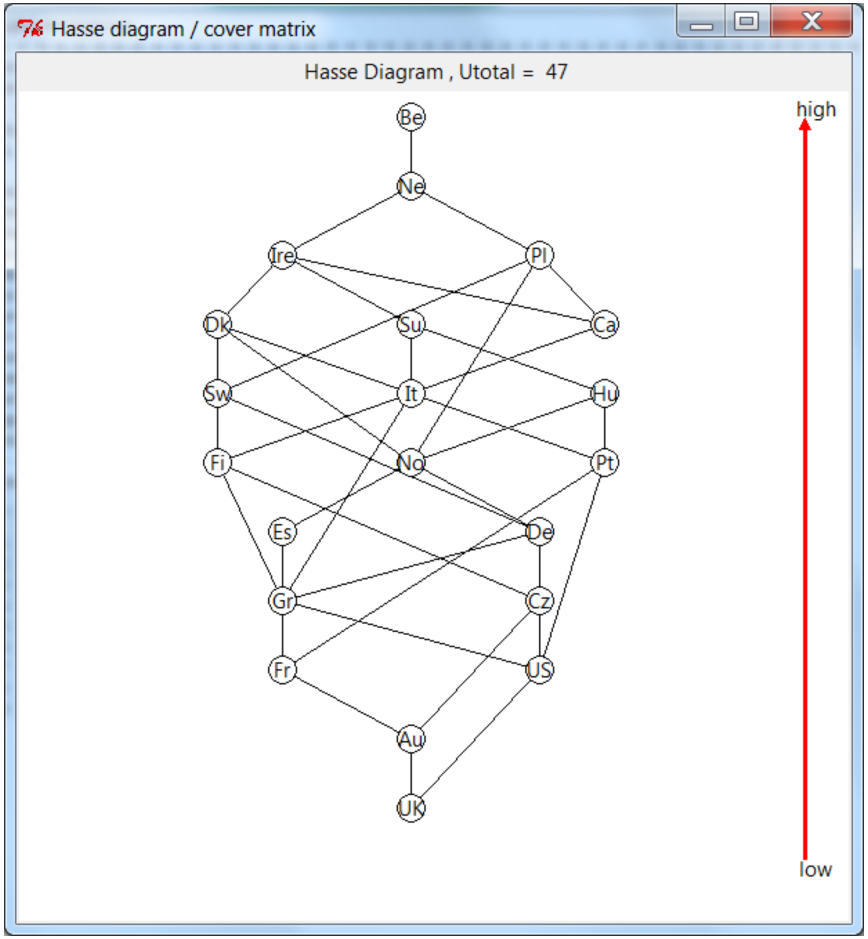
Gr	0.25	0.5
Pl	0.9	0.35
Cz	0.6	0.1
Fr	0.15	0.45
Pt	0.0	0.95
Au	0.1	0.25
Hu	0.4	0.75
US	0.45	0.05
UK	0.2	0.0

Results

g1=0.1, s = 0.3, MC = 1000



g1= 0.5, s=0.3, MC = 1000



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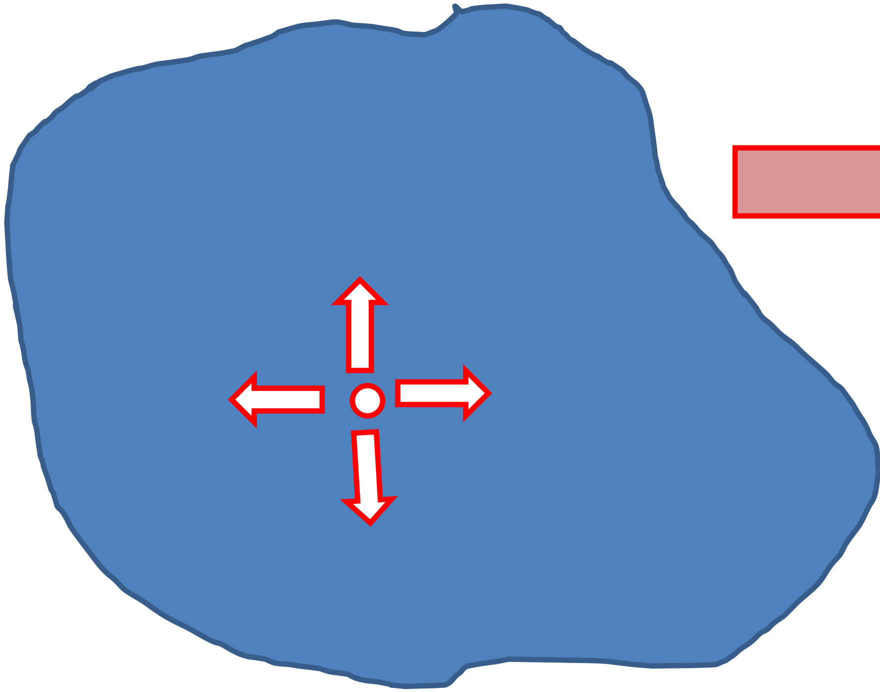
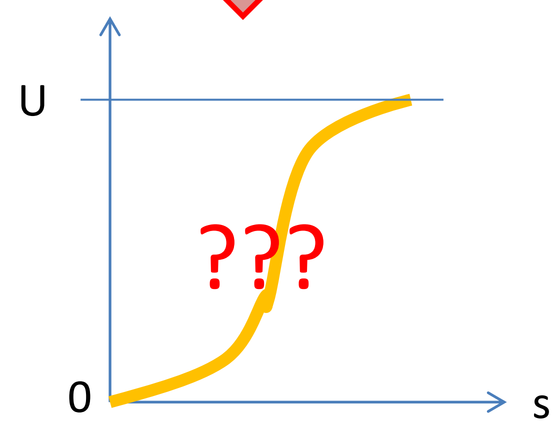


s(j) = |Δ g(j)|,

assumption: s(1) = s(2) =…=s(m) = s

s:=Uncertainty
around **g**



U = f(s) ??