Evaluations as Sets over Lattices

Application point of view

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Bruggemann_Neuchatel_FCA.pptx

Recall: Lecture of Kerber

$$\tilde{\tau}(\alpha, \beta) = \bigvee \{\gamma \mid \tau(\alpha, \gamma) \leq \beta\}.$$

In this case τ is called a *residual* t–norm.

— *This yields a logic* corresponding to L and τ, namely $\tilde{\tau}$.

— o *has* attribute a if and only if $\mathcal{E}(o, a) > 0$. And we put

$$\mathcal{A}'(o) = \tilde{\tau}(\mathcal{A} \Rightarrow \mathcal{E}) = \bigwedge_{a \in A} \tilde{\tau}(\mathcal{A}(a), \mathcal{E}(o, a)).$$

Recall..., cont'd

— We evaluate '$\mathcal{A} \in L^A$ implies $\mathcal{B} \in L^A$ in $\mathcal{E}$' by:

$$\tilde{\tau}(\mathcal{A} \Rightarrow \mathcal{B}) = \bigwedge_{o \in O} \tilde{\tau}(\mathcal{A}'(o), \mathcal{B}'(o)).$$

The focus of this lecture: „implication“ and to reveal the secrets behind mapping $\mathcal{A}$

We are going to apply this mathematical concept

- His: $\mathcal{A}$: A an m-tuple $\{0,1\}^m$
  His: $\mathcal{B}$: B another m-tuple $\{0,1\}^m$
  His: $\tilde{\tau}$: s* (residuum of standard norm)

- $s^*(x,y) = 1$ if $x \leq y$
  $s^*(x,y) = y$ otherwise

Cont‘d

- Q: the indicator set {q(1),…,q(m)}
  X: the set of objects {x(1), x(2),…,x(n)}
- x(i,j) is what Kerber called ε(o,a), i.e. an entry of the data matrix:
  i_{th} object,
  j_{th} indicator

Notation, cont‘d

- In the application we have in mind: A(j), B(j) are selecting certain (crisp) subsets of Q
- I.e.: We want to know whether or not, for instance, q(j) implies q(j*)
- Or more generally:
  {q(j1), q(j2)} implies {q(j3), q(j4)}, etc.

What do we want to know?

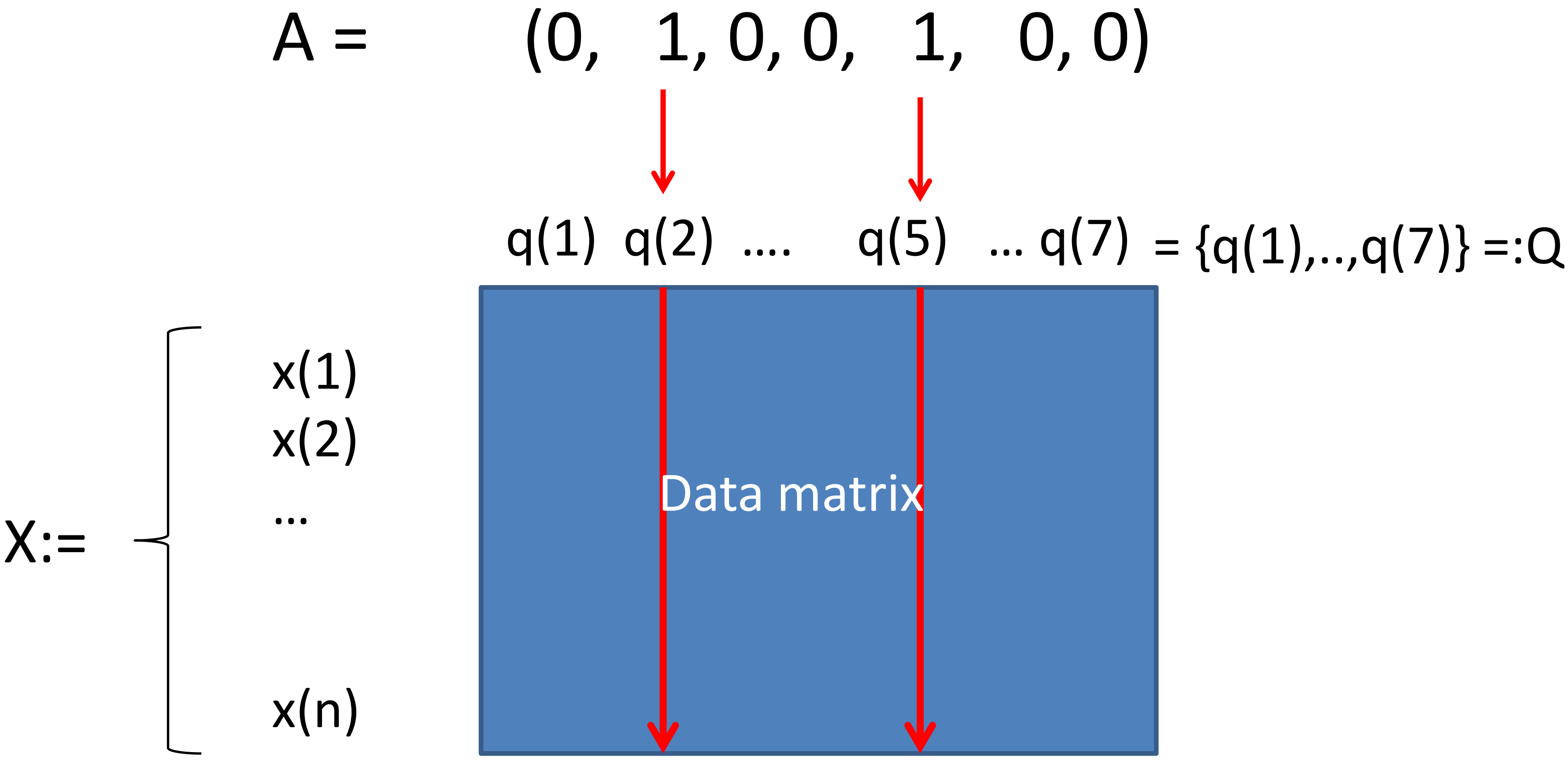
1. How is this simplest question (q(j) ➔ q(j*)) related to the entries of the data matrix?
2. What is the truth value (tv) of this implication
3. And especially: When tv = 1 and what is its meaning in terms of data exploration

First step

- Whether or not an implication holds, depends on the evaluation of the „object x has indicator q(j)“ relation
- Central there is A and its derivation A‘
- A‘(x) needs the calculation of s*
- s*, the residuum of standard norm

For one object x(i) and e.g. A=(0,0,1,0)

- Min{s*(0, x(i,1)), s*(0, x(i,2)),
  s*(1, x(i,3)), s*(0, x(i,4))}
- A‘(x(i)) = Min{1, 1, x(i,3), 1} = x(i,3)
- For example A = (0,1,0,0,1,0,0) would select the 2[nd] and 5[th] indicator of Q, with |Q| = 7



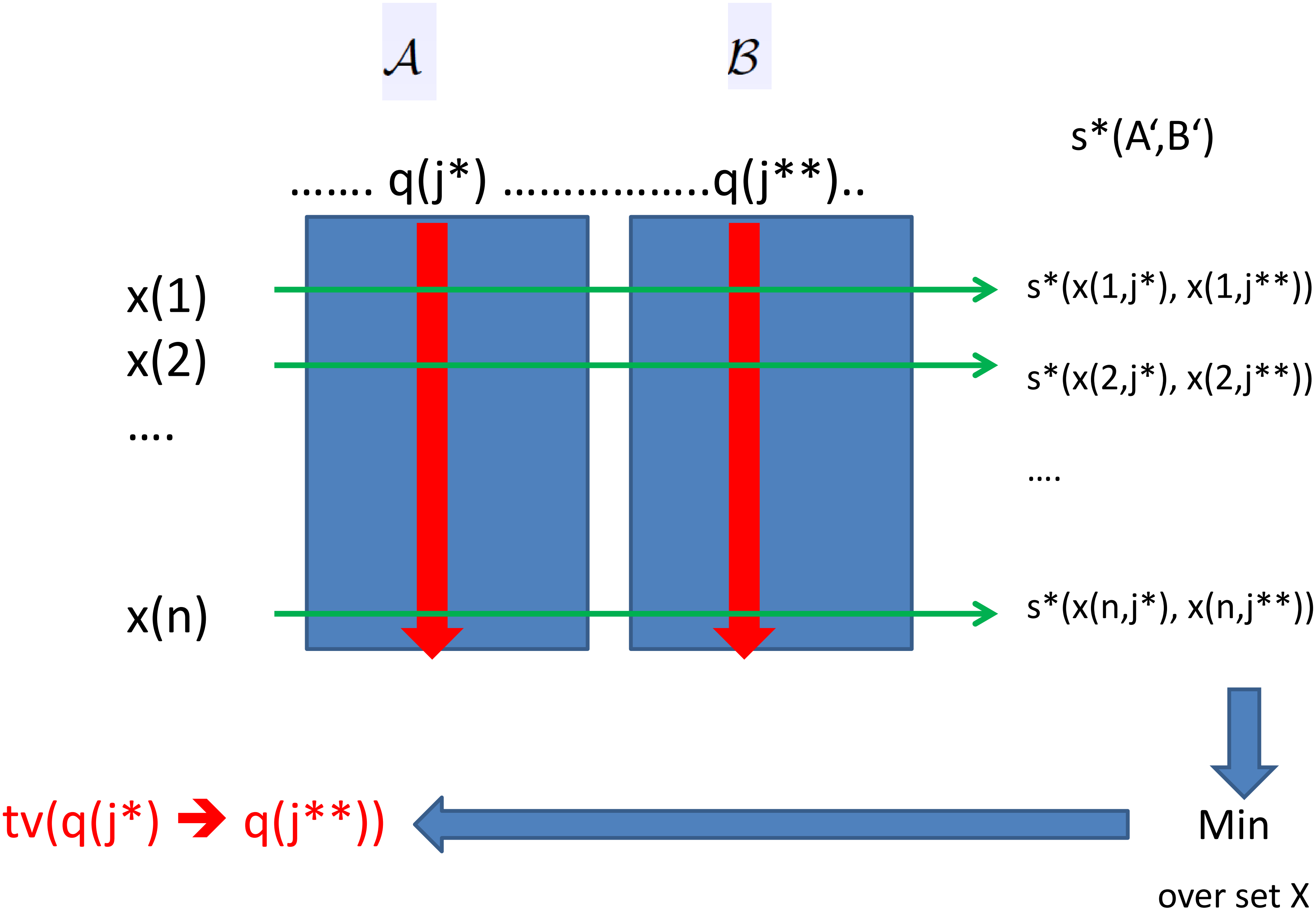
I.e.

(1) For one object x(i), just the values x(i,2) and x(i,5)

(2) Selecting the minimal value for each row

When A describes a singleton {q(j*)}, selecting the j*th indicator in position j* , then the result is x(i,j*).

The evaluation of tv(q(j*) → q(j**)) is now easy:



𝒜
ℬ
s*(A‘,B‘)
……. q(j*) ……………..q(j**)..
x(1)
x(2)
….
x(n)
s*(x(1,j*), x(1,j**))
s*(x(2,j*), x(2,j**))
….
s*(x(n,j*), x(n,j**))
Min
over set X
tv(q(j*) → q(j**))

Example 1: Application of Kerber: The refrigerants

- ALT: atmospheric lifetime
- ODP: Ozone depletion potential
- GWP: General Warming Potential
- Chemical structure (only 3 terms)
  - Cl: presence of Chlorine
  - F: presence of Fluorine
  - nC: At least one C-C bond

An application on Refrigerants, see Kerber: Fuzzy-FCA

PyHasse program L_eval19:

Actually used data matrix

	ALT	ODP	GWP	nC	Cl	F
“1”	0.01	0.2	0.32	0.0	1.0	1.0
“2”	0.03	0.16	0.72	0.0	1.0	1.0
“6”	0.0	0.02	0.05	1.0	1.0	1.0
“7”	0.01	0.01	0.15	1.0	1.0	1.0

standard-norm

premises only by one attribute

Analysis

concerning the set chemicals “1”, “2”, “6”, “7”:

CCl_3F, CCl_2F_2, $C_2H_3Cl_2F$, $C_2H_2ClF_2$

(1) F, implies Cl, with truth-value 1.0
(2) Cl, implies F, with truth-value 1.0
(3) nC, implies F, with truth-value 1.0
(4) nC, implies Cl, with truth-value 1.0
(5) nC, implies Cl, F, with truth-value 1.0
GWP, implies F, with truth-value 1.0
GWP, implies Cl, with truth-value 1.0
GWP, implies Cl, F, with truth-value 1.0
ODP, implies F, with truth-value 1.0
ODP, implies Cl, with truth-value 1.0
ODP, implies Cl, F, with truth-value 1.0
ODP, implies GWP, with truth-value 1.0
ODP, implies GWP, F, with truth-value 1.0
ODP, implies GWP, Cl, with truth-value 1.0
ALT, implies F, with truth-value 1.0
ALT, implies Cl, with truth-value 1.0
ALT, implies GWP, with truth-value 1.0

These results are obtained with data ∈ [0,1] and: restriction on a subset of the first four compounds

a) What is the meaning of truth-value

b) Which role plays the restriction on a certain subset.

CCl_3F, CCl_2F_2, $C_2H_3Cl_2F$, $C_2H_2ClF_2$ → Implic. (1)-(5) trivial

Example 2: Eight regions (labelled 1,10,24,…) along river Rhine.
Pollution of the herb layer by Pb, Cd, Zn and S

standard-norm

premises and conclusions: only one indicator

Analysis

concerning the set of objects as follows

X= {1, 10, 24, 31, 19, 43, 52, 56},

S, implies Zn, with truth-value 0.0

S, implies Cd, with truth-value 0.0

S, implies Pb, with truth-value 0.0

Zn, implies S, with truth-value 0.0

Zn, implies Cd, with truth-value 0.091

Cd, implies Zn, with truth-value 0.476

….

The truth values (tv) are rarely = 1, therefore the questions reformulated:

(1) Under which conditions tv = 1

(2) Can we explore the role of subsets of X?

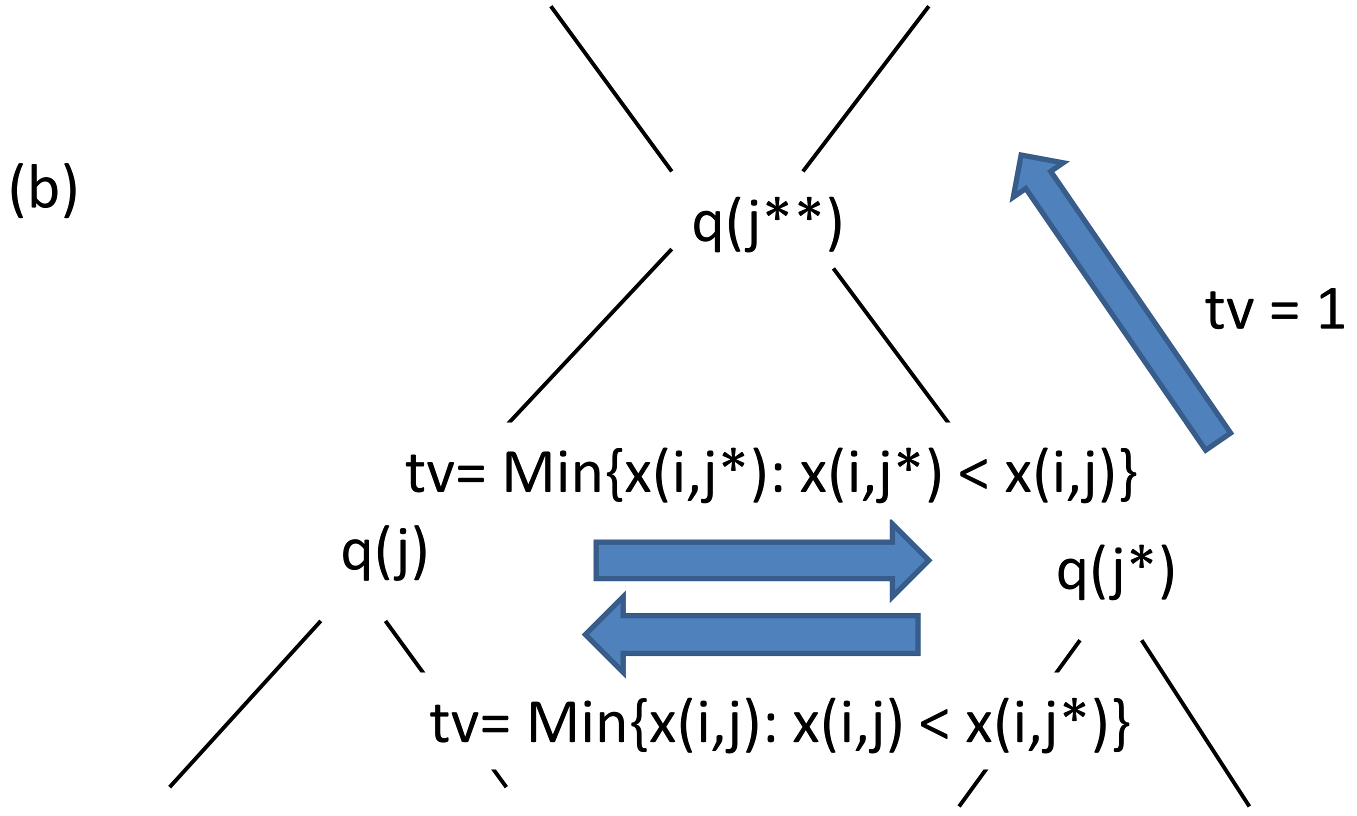
Some observations

(a) For any subset XS of X:

$XS \subseteq X$: $tv(XS) \geq tv(X)$

(b) The product order taken from the transposed data matrix (indicators evaluated by the objects) is relevant:

Observations (cont'd)



Any combinations of indicators: Search their min-value for all x and locate it in the HD of the transposed data matrix

Discussion

- Up to know: Only implications of a special form, namely implications between indicatorsubsets of only one element, are examined in details
- $x(i,j) \leq x(i,j^*)$ for all $i \Rightarrow tv(j \rightarrow j^*) = 1$
- tv and correl seem to have nothing to do with each other
  - tv not symmetric, correl: symmetric
  - if not $x(i,j) \leq x(i,j^*)$ for all i, then tv depends on the smallest value (either of $x(i,j)$ or $x(i,j^*)$)
  - No robustness of tv

Fictitious example

	q1	q2
x1	0	0
x2	0.1	0.1
x3	0.2	0.2
x4	0.3	0.3
x5	0.4	0.4
x6	0.5	0.5
x7	0.6	0.6
x8	0.7	0.7
x9	0.8	0.8
x10	0.9	0.9
x11	1.0	varied

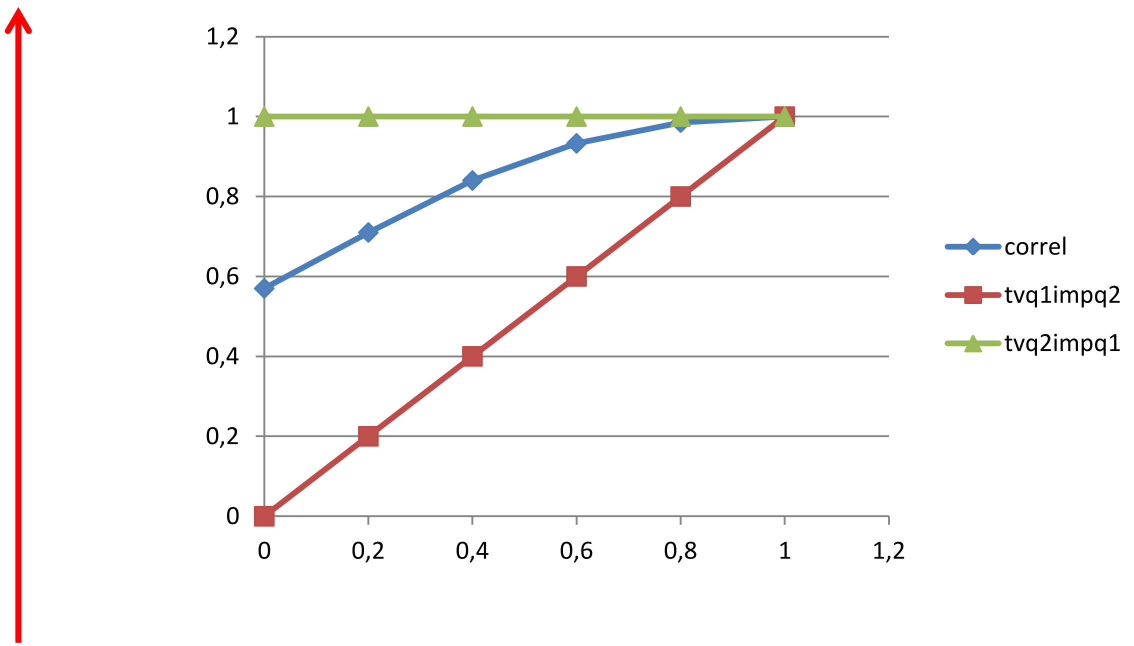
Pearson correlation and tv(q1➔q2) when „varied“ $\in \{0.1, 0.2, \ldots, 1\}$

Correlation vs implication

Correlation: blue

tv(q1➔q2): brown

tv(q2➔q1): green



Answers (take home message)

1. Whether or not $q(j)$ implies $q(j^*)$ depends to the frequency of $x(i,j^*) > x(i,j)$ $x(i) \in XS \subseteq X$
2. tv = 1 if $x(i,j) \leq x(i,j^*)$ for all $x(i) \in XS \subseteq X$
3. tv (of $x \in X$) $\leq$ tv (of $x \in XS \subseteq X$)
4. Correlation and tv seem to be not related

Tasks for the future

- Which role plays the data precision
- Can we find some kind of defuzzification for tv? I.e. As to how far we can see an implication as „relevant“, when tv <1?
- Some work is already done, but is not presented in this lecture, because still many theoretical questions are open:
  - Concepts
  - Implications among subsets of Q, being no singletons
  - Duquenne, Guigues-basis
  - Implications derived directly from concepts (as is possible in the conventional FCA (Ganter, Wille, 1996))

Thank you for attention