

Rigidity of Riemannian foliations with complex leaves on Kähler manifolds *

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Abstract

We study Riemannian foliations with complex leaves on Kähler manifolds. The tensor T , the obstruction to the foliation be totally geodesic is interpreted as a holomorphic section of a certain vector bundle. This enables us to give classification results when the manifold is compact.

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1 Introduction

Riemannian foliations with totally geodesic leaves and in particular Riemannian submersions with totally geodesic fibers are now quite well understood. Many general structure results in the theory of Riemannian submersions are known (see [3], chapter 9) and for particular spaces as spheres or complex and quaternionic projective spaces classification results are available [6, 5] under some geometric hypothesis on the fibers. In the less explored case of pseudo-Riemannian submersions similar results are known to hold under some additional conditions [15, 2, 1]. In the case of Riemannian foliations transversal geometric assumptions were used in order to obtain classification theorems [17]. Note that in most of these works the total geodesicity seems to be an a priori condition.

In a complex setting, a notion of almost Hermitian submersions was proposed in [21] but it turns out that for many classes the horizontal distribution has to be integrable [21, 7]. A less rigid situation, even in the case of a submersion, should arise from the study of Riemannian submersions from an almost-Hermitian manifold. The geometric condition we need here is that the fibers (or the leaves) to be almost complex. This

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is of interest when searching geometric structures admitting a (Riemannian) twistor construction as explained in [?].

In this paper we study Riemannian foliations with complex leaves on Kähler manifolds. The totally geodesic case was completely described in [16], where it is showed that under the simple connectivity and completeness assumptions such an object is a Riemannian product of twistor spaces over positive quaternionic Kähler manifolds, Kähler manifolds and homogeneous spaces belonging to three main classes (see [19] for basic quaternionic-Kähler geometry). Note that for the case of the complex projective space this was already known in [6].

It is then natural to investigate the non-totally geodesic case. It turns out the ambient Kähler geometry is sufficiently strong to force, at least in the compact case, the foliation to be of very special type. More precisely, our main result is the following rigidity theorem.

Theorem 1.1 *Let (M, g, J) be compact Kähler manifold. If M carries a Riemannian foliation \mathcal{F} with complex leaves then M is locally isometric and biholomorphic with a Riemannian product $M_1 \times M_2$ of Kähler manifolds where M_1 carries a totally geodesic, Riemannian foliation with complex leaves and M_2 carries a Riemannian foliation with complex leaves which is transversally integrable. Moreover, the foliation \mathcal{F} is the Riemannian product of the latter.*

As it is well known, the decomposition theorem of deRham ensures that at least locally one can restrict attention to holonomy irreducible Riemannian manifolds. For the case of the latter, theorem 1.1 gives :

Corollary 1.1 *On a compact, simply connected, irreducible Kähler manifold any Riemannian foliation with complex leaves is either totally geodesic, or transversally integrable.*

Note that for these rigidity results no assumption on the curvature of the metric g is necessary. In a standard fashion, conditions ensuring total geodesicity of a given foliation are based on bounds on, say, Ricci curvature (see [12] for examples of results of this type). Note also that when studying holomorphic distributions on Kähler manifolds conditions on the metric are necessary even in the case of (real) codimension 2 [14].

The paper is organized as follows. In section 2 we collect some classical facts about Riemannian foliations and then specialize to the case of Kähler manifolds in section 2. We are basically starting from O'Neill's equations for the curvature tensor and use the Kähler structure to derive differential relations between the basic tensors A and T . In section 3 we interpret the tensor T , the obstruction to the foliation to be totally geodesic as a holomorphic section of a certain vector bundle and use the compactness of M in order to obtain the splitting in theorem 1.1.

2 Preliminaries

We start by collecting a number of basic facts about Riemannian foliations and next we will specialize to the Kähler case. Let (M, g) be a Riemannian manifold and let \mathcal{F}

be a foliation on M . We denote by \mathcal{V} the integrable distribution induced by \mathcal{F} . Let H be the orthogonal complement of \mathcal{V} . We assume the foliation \mathcal{F} to be Riemannian, that is

$$\mathcal{L}_V g(X, Y) = 0$$

whenever X, Y are in H and V belongs to \mathcal{V} . Let ∇ be the Levi-Civita connection of the metric g . Throughout this paper we will denote by V, W vector fields in \mathcal{V} and by X, Y, Z etc. vector fields in H . It is easy to verify that the formula [18]

$$\overline{\nabla}_E F = (\nabla_E F_{\mathcal{V}})_{\mathcal{V}} + (\nabla_E F_H)_H$$

defines a metric connection with torsion on M (here the subscript denotes orthogonal projection on the subspace). The main property of this connection is that it preserves the distributions \mathcal{V} and H . If T and A are the O'Neill's tensors of the foliation then the following relations between ∇ and $\overline{\nabla}$ are known to hold

$$\begin{aligned} \nabla_X Y &= \overline{\nabla}_X Y + A_X Y, & \nabla_X V &= \overline{\nabla}_X V + A_X V \\ \nabla_V X &= \overline{\nabla}_V X + T_V X, & \nabla_V W &= \overline{\nabla}_V W + T_V W. \end{aligned}$$

For the algebraic properties of T and A see [18]. We only recall here that A is skew-symmetric on H while T is symmetric on \mathcal{V} .

In the rest of this paper we will assume that (M, g) is a Kähler manifold of dimension $2m$, with complex structure J . Moreover, we suppose that the foliation \mathcal{F} has complex leaves, that is $J\mathcal{V} = \mathcal{V}$ (then of course, $JH = H$). As $\nabla J = 0$, it follows that $\overline{\nabla} J = 0$, hence we obtain information about the complex type of the tensors A and T as follows

$$\begin{aligned} \mathbf{2.1} \quad A_X(JY) &= J(A_X Y), & A_{JX} V &= -J(A_X V) = A_X(JV) \\ T_{JV} W &= J(T_V W), & T_{JV} X &= -J(T_V X) = T_V(JX). \end{aligned}$$

We also have $A_{JX} JY = -A_X Y$ and $T_{JV} JW = -T_V W$. A consequence of the last identity is that the foliation \mathcal{F} is harmonic, that is the mean curvature vector field vanishes.

We will use now the Kähler structure on M , together with suitable curvature identities to get some geometric information about the tensors A and T .

Lemma 2.1 *Let X, Y, Z be in H and V, W in \mathcal{V} . Then we have :*

- (i) $(\overline{\nabla}_X A)(Y, Z) = 0$
- (ii) $\langle A_X Y, T_V Z \rangle = 0$
- (iii) $\langle (\overline{\nabla}_V A)(X, Y), W \rangle = \langle (\overline{\nabla}_W A)(X, Y), V \rangle$.
- (iv) $(\overline{\nabla}_{JX} T)(V, W) = -J(\overline{\nabla}_X T)(V, W)$

Proof :

We will prove (i) and (ii) simultaneously. Let us denote by R the curvature tensor of the Levi-Civita connection of the metric g . We first recall the O'Neill formula (see [18])

$$\begin{aligned} \mathbf{2.2} \quad R(X, Y, Z, V) &= \langle (\overline{\nabla}_Z A)(X, Y), V \rangle + \langle A_X Y, T_V Z \rangle - \\ &\langle A_Y Z, T_V X \rangle - \langle A_Z X, T_V Y \rangle. \end{aligned}$$

Since (M, g) is Kähler one has $R(JX, JY, Z, V) = R(X, Y, Z, V)$. Hence by (1.1) we easily arrive at $\langle (\bar{\nabla}_X A)(Y, Z), V \rangle + \langle A_X Y, T_V Z \rangle = 0$. But we know that (see [18], page 52)

$$\sigma_{X,Y,Z} \langle (\bar{\nabla}_X A)(Y, Z), V \rangle = \sigma_{X,Y,Z} \langle A_X Y, T_V Z \rangle$$

(here σ denotes the cyclic sum) thus $\sigma_{X,Y,Z} \langle A_X Y, T_V Z \rangle = 0$ and further $R(X, Y, Z, V) = \langle A_X Y, T_V Z \rangle$. Using again the J -linearity of R we get immediately (ii) hence (i) follows.

To prove (iii) we use another O'Neill's formula stating that

$$\begin{aligned} \mathbf{2.3} \quad R(V, W, X, Y) = & \langle (\bar{\nabla}_V A)(X, Y), W \rangle - \langle (\bar{\nabla}_W A)(X, Y), V \rangle \\ & + \langle A_X V, A_Y W \rangle - \langle A_X W, A_Y V \rangle - \langle T_V X, T_W Y \rangle + \langle T_V X, T_W Y \rangle . \end{aligned}$$

The result follows now by (1.1) and the fact that $R(V, W, JX, JY) = R(V, W, X, Y)$. The identity in (iv) can be proven in the same way, using this time the identity

$$\begin{aligned} R(X, V, Y, W) = & \langle (\bar{\nabla}_X T)(V, W), Y \rangle + \langle (\bar{\nabla}_V A)(X, Y), W \rangle \\ & + \langle A_X V, A_Y W \rangle - \langle T_V X, T_W Y \rangle \end{aligned}$$

the fact that $R(JX, JV, Y, V) = R(X, V, Y, W)$ and (iii) ■

Remark 2.1 (i) *By the the first two assertions of lemma 2.1 we obtain that $R(X, Y, Z, V) = 0$, a condition frequently imposed when studying Riemannian foliations (see chapter 5 of [18] and references therein).*

(ii) *By (i) and (ii) of the previous lemma it is easy to see that H satisfies the Yang-Mills condition.*

(iii) *Using (iii) of Proposition 2.1 and [18], page 52, we get the following relation between the covariant derivatives of A and T*

$$\mathbf{2.4} \quad 2 \langle (\bar{\nabla}_V A)(X, Y), W \rangle = \langle (\bar{\nabla}_Y T)(V, W), X \rangle - \langle (\bar{\nabla}_X T)(V, W), Y \rangle .$$

We will make use of this in the next section.

Let us denote by \bar{R} the curvature tensor of the connection $\bar{\nabla}$. Another result that will be needed in the next section is the following.

Lemma 2.2 *We have :*

$$\bar{R}(X, Y)V = 2[A_X, A_Y]V + Q(X, Y)V$$

for all X, Y in H and V in \mathcal{V} where we defined $Q(X, Y)V = T_{TV}X - T_{TX}Y$.

The proof follows from the general formulas in [18], page 100, and lemma 2.1, (iii).

3 The harmonicity of the tensor T

In this section we begin the study of the tensor T . Our main idea is to consider T as a $S^2(\mathcal{V})$ -valued 1-form on M and then use lemma 2.1, (iv) to study differential equations involving T . The analogy we have constantly in mind is the well known fact that on a compact Kähler manifold any holomorphic 1-form is closed. We develop first some preliminary material. We refer the reader to the discussion in section 4 of [4]. Although our geometric context is different the guidelines principle concerning

the Kähler identities and relations between various natural differential operators is the same.

For each $p \geq 0$ we define $S^2(\mathcal{V}) \otimes \Omega^p(H)$ to be the space of symmetric endomorphisms $\alpha : \mathcal{V} \times \mathcal{V} \rightarrow \Omega^p(H)$. We also define $S_A^2(\mathcal{V})$ as the subspace of $S^2(\mathcal{V})$ consisting of tensors which vanish on $A_X Y, X, Y$ in H .

The ordinary exterior derivative d does not preserve $\Lambda^*(M)$ but d_H , the horizontal component of its restriction to $\Lambda^*(H)$ does. The latter can be extended to $S^2(\mathcal{V}) \otimes \Lambda^*(H)$ by setting

$$(d_H \alpha)(V, W)(X_0, \dots, X_p) = \sum_{i=0}^p (-1)^i (\bar{\nabla}_{X_i} \alpha)(V, W)(X_0, \dots, \hat{X}_i, \dots, X_p)$$

for every α in $S^2(\mathcal{V}) \otimes \Lambda^p(H)$. Using lemma 2.1, (ii) it is easy to see that d_H preserves $S_A^2(\mathcal{V}) \otimes \Lambda^p(H)$. The fact that the almost complex structure J is integrable induces a splitting

$$d_H = \partial_H + \bar{\partial}_H$$

on $S^2(\mathcal{V}) \otimes \Lambda^*(H)$ where $\partial_H : S^2(\mathcal{V}) \otimes \Lambda^{p,q}(H) \rightarrow S^2(\mathcal{V}) \otimes \Lambda^{p,q+1}(H)$ and $\bar{\partial}_H : S^2(\mathcal{V}) \otimes \Lambda^{p,q}(H) \rightarrow S^2(\mathcal{V}) \otimes \Lambda^{p+1,q}(H)$.

We need now a formula relating to the anticommutator of the operators ∂_H and $\bar{\partial}_H$. Let Q be the tensor defined at the end of section 1. If s belongs to $S_A^2(\mathcal{V})$ we define the action of Q on s to be $Q.s$, an element of $S_A^2(\mathcal{V}) \times \Lambda^2(H)$ defined by $(Q.s)(V, W)(X, Y) = s(Q(X, Y)V, W) + s(V, Q(X, Y)W)$. Obviously, this can be extended to give a linear application

$$\mathcal{P} : S_A^2(\mathcal{V}) \otimes \Lambda^p(H) \rightarrow S_A^2(\mathcal{V}) \otimes \Lambda^{p+2}(H), \mathcal{P}\alpha = Q.\alpha$$

having the property that $\mathcal{P}(s\alpha) = Q.s \wedge \alpha$ whenever s is in $S_A^2(\mathcal{V})$ and α belongs to $\Lambda^p(H)$.

Lemma 3.1 *The following holds on $S_A^2(\mathcal{V}) \otimes \Lambda^{p,q}(H)$:*

$$\partial_H \bar{\partial}_H + \bar{\partial}_H \partial_H = \mathcal{P}$$

Proof :

Let us first compute $d_H^2 q$ where q belongs to $S_A^2(\mathcal{V})$. An easy manipulation yields

$$(d_H^2 q)(V, W)(X, Y) = (\bar{\nabla}_{X,Y}^2 q)(V, W) - (\bar{\nabla}_{Y,X}^2 q)(V, W).$$

Using the Ricci identity for the connection with torsion $\bar{\nabla}$ (see [3], page 26) we get

$$(\bar{\nabla}_{X,Y}^2 q)(V, W) - (\bar{\nabla}_{Y,X}^2 q)(V, W) = q(\bar{R}(X, Y)V, W) + q(V, \bar{R}(X, Y)W) + 2(\bar{\nabla}_{A_X Y} q)(V, W).$$

Using now lemma 2.2 and the fact that q vanishes on vectors of the form $A_X Y$ with X, Y in H we obtain that

$$\mathbf{3.1} \quad (d_H^2 q)(V, W)(X, Y) = 2(\bar{\nabla}_{A_X Y} q)(V, W) + q(Q(X, Y)V, W) + q(V, Q(X, Y)W).$$

We consider now α in $S_A^2(\mathcal{V}) \otimes \Lambda^{p,q}(H)$ and let $\{e^I\}$ a local basis of basic (p, q) -forms in $\Lambda^{p,q}(H)$. We write $\alpha = \sum_I q_I e^I$ with q_I in $S_A^2(\mathcal{V})$. Since d_H^2 vanishes on basic forms

(see [18]) we get using the multiplicative properties of d_H that $d_H^2\alpha = \sum_I d_H^2 q_I \wedge e^I$. But $\partial_H \bar{\partial}_H \alpha + \bar{\partial}_H \partial_H \alpha = (d_H^2 \alpha)^{p+1, q+1} = \sum_I (d_H^2 q_I)^{1,1} \wedge e^I$. But $A_{JX} JY = -A_X Y$ and $Q(JX, JY) = Q(X, Y)$ hence $(d_H^2 q_I)^{1,1} = \mathcal{P} q_I$ and the proof is finished ■

At this stage let us recall another particular feature of Kähler geometry, namely the Kähler identities. We state them on $\Lambda^p(H)$ as follows :

$$\begin{aligned} [\partial_H, L^*] &= -i\bar{\partial}_H^*, & [\bar{\partial}_H, L^*] &= i\partial_H^* \\ [\partial_H^*, L] &= -i\bar{\partial}_H, & [\bar{\partial}_H^*, L] &= i\partial_H \end{aligned}$$

where L is multiplication with the two form ω^H in $\Lambda^2(H)$ defined by $\omega^H(X, JY) = \langle X, JY \rangle$. Of course these are projection of the Kähler identities of M and furthermore it is easy to see that they hold on $S_A^2(\mathcal{V}) \otimes \Lambda^*(H)$ too.

Let us now define ζ in $S_A^2(\mathcal{V}) \otimes \Lambda^{0,1}(H)$ by $\zeta = \alpha_T + iJ\alpha_T$. Then

Lemma 3.2 (i) $\bar{\nabla}_{JX}\zeta = -i\bar{\nabla}_X\zeta$ for all X in H

(ii) $\bar{\partial}_H\zeta = 0$

(iii) $\partial_H^*\zeta = \bar{\partial}_H^*\zeta = 0$

(iv) $\partial_H^*\bar{\partial}_H\zeta = -i\mathcal{P}^*L\zeta$.

Proof :

(i) is a straightforward consequence of lemma 2.1, (iv), while (ii) comes immediately by (i) and the fact that $2\bar{\partial}_H = d_H + iJd_HJ$ on $S_A^2(\mathcal{V}) \otimes \Lambda^{0,1}(H)$.

To prove (iv) we use (in the classical way) the Kähler identities and the previous lemma. We have

$$\begin{aligned} \partial_H^*\bar{\partial}_H\zeta &= -i\partial_H^*[\bar{\partial}_H^*, L]\zeta = -i(\partial_H^*\bar{\partial}_H^*)L\zeta = -i\{\partial_H^*, \bar{\partial}_H^*\}L\zeta + i\bar{\partial}_H^*[\partial_H^*, L]\zeta = \\ &= -i\{\partial_H^*, \bar{\partial}_H^*\}L\zeta + \bar{\partial}_H^*\bar{\partial}_H\zeta = -i\{\partial_H^*, \bar{\partial}_H^*\}L\zeta. \end{aligned}$$

It suffices now to dualize the equation in lemma 3.1 ■

Before proceeding to the proof of the theorem 1.1 we need one more preliminary result.

Lemma 3.3 We have $\mathcal{P}\zeta = 0$.

Proof :

Obviously it suffices to show that $\mathcal{P}\alpha_T = 0$. But it is straightforward to see that

$$\begin{aligned} (\mathcal{P}\alpha)(V, W)(X, Y, Z) &= \alpha(Q(X, Y)V, W)(Z) + \alpha(V, Q(X, Y)W)(Z) \\ &\quad - \alpha(Q(X, Z)V, W)(Y) - \alpha(V, Q(X, Z)W)(Y) \\ &\quad + \alpha(Q(Y, Z)V, W)(X) + \alpha(V, Q(Y, Z)W)(X) \end{aligned}$$

whenever α belongs to $S_A^2(\mathcal{V}) \otimes \Lambda^1(H)$. We have :

$$\begin{aligned} \langle T_V Q(X, Y)W, Z \rangle &= - \langle Q(X, Y)W, T_V X \rangle = - \langle T_{T_W Y} X - T_{T_W X} Y, T_V Z \rangle = \\ &= \langle X, T_{T_V Z} (T_W Y) \rangle - \langle Y, T_{T_W X} T_V Z \rangle. \end{aligned}$$

But $\langle X, T_{T_V Z} (T_W Y) \rangle = - \langle T_{T_V Z} X, T_W Y \rangle = \langle T_W T_{T_V Z} X, Y \rangle$ hence

$$\langle T_V Q(X, Y)W, Z \rangle = \langle T_W T_{T_V Z} X, Y \rangle - \langle T_{T_W X} T_V Z, Y \rangle.$$

Taking the alternate sum on X, Y, Z of this formula gives now easily the result ■

Let we assume, in the rest of this section, that the manifold M is compact and then prove theorem 1.1. At first, taking the scalar product with ζ in lemma 3.2, (iv) and integrating over M we obtain by lemma 3.3 that $\partial_H \zeta = 0$ and since $\bar{\partial}_H \zeta = 0$ it follows that $d_H \zeta = 0$ and further $d_H \alpha_T = 0$. Using now (2.4) we obtain that $(\bar{\nabla}_V A)(X, Y) = 0$ and we conclude that

$$\mathbf{3.2} \quad (\bar{\nabla}_E A)(X, Y) = 0$$

for all E in TM . But the fact that $d_H \alpha_T = 0$ still contains usefull information. We proceed as follows.

Using the proof of lemma 3.1 (namely formula (3.1) and expression in a local basis of basic forms), one obtains after a few standard manipulations :

$$\mathbf{3.3} \quad d_H^2 = 2\mathcal{L} + \mathcal{P}$$

on $S_A^2(\mathcal{V}) \otimes \Lambda^1(H)$ where

$$(\mathcal{L}\alpha)(V, W)(X, Y, Z) = (\bar{\nabla}_{A_Y Z} \alpha)(V, W)(X) - (\bar{\nabla}_{A_X Z} \alpha)(V, W)(Y) + (\bar{\nabla}_{A_X Y} \alpha)(V, W)(Z) + \alpha(V, W)(A_X A_Y Z - A_Y A_X Z + A_Z A_X Y)$$

Remark 3.1 *Formulas of type (3.3) can be proven for forms of any degree and the operator \mathcal{L} can be given a more concise form. Since only the case of 1-forms is needed for our purposes this presentation makes more visual subsequent computations.*

Lemma 3.4 $A_X(T_V W) = 0$.

Proof :

Let us recall first the following O'Neill formula :

$$R(V_1, V_2, V_3, Z) = \langle (\bar{\nabla}_{V_2} T)(V_1, V_3), Z \rangle - \langle (\bar{\nabla}_{V_1} T)(V_2, V_3), Z \rangle .$$

Now, by lemma 2.1, (ii) and (3.1) we get $\langle (\bar{\nabla}_{V_1} T)(A_X Y, V_3), Z \rangle = 0$ and it follows that $R(V_1, A_X Y, V_3, Z) = \langle (\bar{\nabla}_{A_X Y} T)(V_1, V_3), Z \rangle$. Since (M, g, J) is Kähler $R(JV_1, A_X(JY), V_3, Z) = R(V_1, A_X Y, V_3, Z)$ which yields further to

$$(\bar{\nabla}_{A_{JX} Y} T)(V_1, V_3) = -J(\bar{\nabla}_{A_X Y} T)(V_1, V_3).$$

Using this and relations (2.1) for the tensor A we obtain after some computations that

$$(\mathcal{L}\alpha_T)(V, W)(JX, JY, Z) + (\mathcal{L}\alpha_T)(V, W)(X, Y, Z) = 2 \langle T_V W, A_X A_Y Z - A_Y A_X Z \rangle .$$

Or the vanishing of $d_H \alpha_T = 0$ and $\mathcal{P}\alpha_T$ implies that of $\mathcal{P}\alpha_T$ hence

$$\langle T_V W, A_X A_Y Z - A_Y A_X Z \rangle = 0$$

for all X, Y, Z in H and V, W in \mathcal{V} . Taking in this last reequation $Y = JX$ we arrive at $\langle A_X(T_V W), A_X(JZ) \rangle = 0$ and the conclusion is straightforward ■

For each m in M we define \mathcal{V}_m^0 to be the vectorial subspace of \mathcal{V}_m spanned by $\{A_X Y : X, Y \text{ in } H_m\}$ and let H_m^0 be the linear span of $\{A_X V : X \text{ in } H_m, V \text{ in } \mathcal{V}_m\}$. By

(3.1) and using parallel transport with respect to the connection $\bar{\nabla}$ we see that we obtained smooth distributions \mathcal{V}^0 and H^0 of TM which are furthermore $\bar{\nabla}$ -parallel. We denote by \mathcal{V}^1 resp. H^1 the orthogonal complement of \mathcal{V}^0 resp. H^0 in \mathcal{V} resp. H . We moreover define distributions $D^i = \mathcal{V}^i \oplus H^i$, $i = 0, 1$ of TM . They are both $\bar{\nabla}$ -parallel because D^0 is and D^0 is orthogonal to D^1 (of course $TM = D^0 \oplus D^1$, an orthogonal direct sum). Moreover, D^0 is ∇ -parallel by lemma 2.1, (ii) and lemma 3.2, (ii). As by the same reasons T resp. A are vanishing on D^1 resp. D^2 the proof of the theorem 1.1 is finished by using the decomposition theorem of DeRham.

References

- [1] G. BADITOIU, *Semi-Riemannian submersions with totally geodesic fibers*, eprint [arXiv:math.DG/0005254](https://arxiv.org/abs/math/0005254)
- [2] G. BADITOIU, S. IANUS, *Semi-Riemannian submersions from real and complex pseudo-hyperbolic spaces*, *Diff. Geom. Appl.* **16** (2002), 79-94.
- [3] L. BÉRARD BERGERY, T. OCHIAI, *On some generalizations of the construction of twistor spaces*, *Proc. Symp., Durham/Engl.* 1982, (1984), 52-59.
- [4] A. BESSE, *Einstein manifolds*, Springer-Verlag, New York, 1987.
- [5] S.K. DONALDSON, *Yang-Mills invariants of four manifolds in Geometry of low-dimensional manifolds I*, *London Math. Soc. Lect. Notes* **150**, (1992), 5-40.
- [6] R. ESCOBALES, *Riemannian submersions with totally geodesic fibers*, *J. Diff. Geom.* **10** (1975), 253-276.
- [7] R. ESCOBALES, *Riemannian submersions from complex projective spaces*, *J. Diff. Geom.* **13** (1978), 253-276.
- [8] M. FALCITELLI, PASTORE A.M. *A note on almost Kähler and nearly Kähler submersions*, *J. Geom.* **69** (2000), 79-87.
- [9] A. GRAY, L. M. HERVELLA, *The sixteen classes of almost Hermitian manifolds and their linear invariants*, *Ann. Mat. Pura Appl.* **123** (1980), 35-58.
- [10] D. L. JOHNSON, *Kähler submersions and holomorphic connections*, *J. Diff. Geom.* **15** (1980), 71-79.
- [11] D. L. JOHNSON, L. B. WHITT, *Totally geodesic foliations*, *J. Diff. Geom.* **15** (1980), 225-235.
- [12] F. KAMBER, PH. TONDEUR, *Curvature properties of harmonic foliations*, *Illinois J. Math.* **18**, (1984), 458-471.
- [13] S. KOBAYASHI, K. NOMIZU, *Foundations of differential geometry*, vol.1, Wiley, 1963.
- [14] C. LEBRUN, *Fano manifolds, contact structures and quaternionic geometry*, *International J. Math.* **6** (1995), 419-437.
- [15] M. MAGID, *Riemannian submersions from anti-de Sitter space with totally geodesic fibers*, *J. Diff. Geom.* **16**, (1981), 323-331.

- [16] P.A.NAGY, *Nearly Kähler geometry and Riemannian foliations*, eprint **arXiv:math. DG/0203038**.
- [17] PH. TONDEUR, L. VANHECKE, *Transversally symmetric Riemannian foliations*, Tôhoku Math. J. **42** (1990), 307-317.
- [18] PH. TONDEUR, *Geometry of Foliations*, Birkhäuser Verlag, 1997.
- [19] S. SALAMON, *Quaternionic Kähler manifolds*, Invent. Math. **67** (1982), 143-171.
- [20] B. WATSON, *Almost hermitian submersions*, J. Diff. Geom. **11** (1976), 147-165.

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