N. Bucalovic,^{1,*} V. Dolgovskiy,¹ M. C. Stumpf,^{1,2} C. Schori,¹ G. Di Domenico,¹ U. Keller,² S. Schilt,¹ and T. Südmeyer^{1,2}

¹Laboratoire Temps-Fréquence, Université de Neuchâtel, 2000 Neuchâtel, Switzerland ²Institute of Quantum Electronics, ETH Zurich, 8093 Zürich, Switzerland *Corresponding author: nikola.bucalovic@unine.ch

We investigate the dynamics of the carrier-envelope-offset (CEO) frequency, f_{CEO} , controlled by a pump current on the self-referencing of an optical frequency comb generated from a diode-pumped solid-state laser at 1.56 µm. We observe a reversal point in the tuning of f_{CEO} with the pump current. Between the low- and high-frequency region in the dynamic response of f_{CEO} to pump current modulation, we observe a significant phase shift of ≈180 deg in the transfer function. As a result, it is impossible to stabilize f_{CEO} at a pump current above the reversal point, although the free-running CEO beat at this point has a higher signal-to-noise ratio than underneath the reversal point at which the locking is straightforward. Our results indicate that a high signal-to-noise ratio and a low-noise CEO beat are not sufficient indicators for the feasibility of comb self-referencing in cases for which CEO dynamics play a dominant role.

In the past decade, fully stabilized optical frequency combs became a key component for many applications, such as optical atomic clocks, ultra-low-noise microwave generation or broadband high-resolution spectroscopy. Among the different existing comb technologies, Ti:Sapphire-based combs were the first to be selfreferenced [1]. This traditional technology is still commonly employed because of its low intrinsic noise and, to date, it constitutes the only laser capable of directly generating the octave-spanning spectrum required for comb self-referencing [2]. Ti:Sapphire lasers, however, suffer from some practical constraints, such as their complexity, high cost, and inefficient pumping. On the other hand, fiber oscillator systems are convenient and robust, but they are intrinsically noisier and thus require the use of more elaborate noise reduction techniques and higher feedback bandwidth to achieve a tight lock of the carrierenvelope-offset (CEO) beat. A promising alternative comb technology is the semiconductor saturable absorber mirror (SESAM)-mode-locked diode-pumped solidstate laser (DPSSL), which combines positive aspects of the other two comb technologies, such as low intrinsic noise, high repetition rate capability, and efficient diode pumping.

A key step in the achievement of a fully stabilized optical frequency comb is the detection of the CEO beat in an f to 2f interferometer [3] and its subsequent phase-lock to a stable rf reference using a feedback loop. The pump power of the femtosecond laser generally is used to control and stabilize the CEO frequency f_{CEO} . This is traditionally accomplished using an acousto-optic modulator in a Ti:Sapphire laser, whereas direct control of the injection current is the standard method applied to diode-pumped lasers, such as fiber lasers and DPSSLs.

Many physical effects contribute to the change of f_{CEO} with the pump power in a frequency comb, as it is exhaustively discussed in Newbury and Washburn [4]. The response of f_{CEO} to a change in the pump power

thus is not simple and generally leads to a nonlinear behavior, which may affect the CEO stabilization. In a Ti:Sapphire comb, the existence of a reversal point in the dependence of $f_{\rm CEO}$ on the pump power has been shown in Holman *et al.* [5], with an important consequence for the CEO stabilization. This reversal point was accompanied by a significant change of the CEO frequency noise, revealing a strong reduction of the linewidth of the freerunning CEO beat around this particular point. As a consequence, a much better stabilization of the CEO beat (i.e., achieving a lower residual phase noise) was obtained by tuning the pump power close to the reversal point, at which the contribution of pump power fluctuations to the CEO noise was minimized.

In this Letter we report on the observation of a reversal point in the variation of f_{CEO} with the pump current in a $1.5 \ \mu m$ DPSSL comb, with a different but crucial impact on the CEO stabilization. We show that the dynamics of $f_{\rm CEO}$, that is, its frequency response to pump current modulation, depends on the position of the operating point relative to the reversal point and plays a major role in the ability to phase-stabilize f_{CEO} . More specifically, the dynamic response changes differently at low and high modulation frequencies on either side of the reversal point, indicating that different effects contribute to the tuning of f_{CEO} with the pump current. The observed dynamic response, in particular in terms of its phase, fully explains the experimentally observed impossibility to stabilize f_{CEO} when the pump current was tuned above the reversal point, whereas a tight phase-lock was straightforwardly achieved below the reversal point.

Our DPSSL comb is generated from a passively modelocked Er:Yb:glass laser oscillator (ERGO), emitting in the 1.56 μ m spectral region with a repetition rate of 75 MHz [6]. A pump diode current control, resulting in a linear variation of the pump power, is implemented as a standard method of f_{CEO} fine-tuning and stabilization. By the direct influence on the pulse duration and energy, the pump current variations translate into a change of the CEO frequency.

The static tuning of f_{CEO} with the pump current is nonlinear and even nonmonotonous in the ERGO comb, as illustrated in Fig. 1. A reversal point at which the slope of the dependence of f_{CEO} on the pump current I_p changes its sign occurs at $I_{\text{reversal}} \approx 860$ mA, corresponding to a pump power of ≈563 mW. In the ERGO comb, the CEO frequency is locked to a 20 MHz reference [which should be achievable in principle at the two different currents labeled I_{low} (*817 mA) and I_{high} (*893 mA), respectively, for which $f_{\text{CEO}} = 20$ MHz (Fig. 1), provided that a correct sign is chosen for the feedback signal applied to the pump current. In practice, this is not the case and a strongly different behavior was observed on both sides of the reversal point. The lock is possible only at the operating current I_{low} located below the reversal point at which the CEO tuning coefficient $\partial f_{\text{CEO}}/\partial I_p$ is positive. Despite the better signal-to-noise ratio of the CEO beat obtained at the higher current I_{high} located above the reversal point (40 dB observed at 100 kHz resolution bandwidth compared with 33 dB below the reversal point, as shown in the inset in Fig. 2), no locking could be achieved at this point.

In contrast to the previous observations made in the Ti:Sapphire laser of Holman *et al.* [5], we did not see any significant change in the frequency noise of the CEO beat, and thus on its free-running linewidth, when tuning the pump current across the reversal point. Figure 2 shows a similar frequency noise PSD of the CEO beat at the two operating currents I_{low} and I_{high} , measured using a frequency discriminator [9] and a fast Fourier transform analyzer. This indicates that the frequency noise of the CEO beat is not dominated by the fluctuations of the pump laser power in our comb and that some other effect is responsible for the inability to lock f_{CEO} at a high current value I_{high} .

To investigate this issue, the dynamic response of f_{CEO} was measured in amplitude and phase in a standard lockin scheme, for a small pump current modulation depth (1 mA peak-to-peak) and using a frequency discriminator (frequency-to-voltage converter) to demodulate the CEO beat. Figure 3 displays the transfer function of f_{CEO}



Fig. 1. (Color online) Static tuning curve of the CEO frequency $f_{\rm CEO}$ with respect to the pump current showing a reversal point at $I_p \approx 860$ mA: $f_{\rm CEO}$ vs I_p (left scale, black circles) and static tuning coefficient (right scale, grey squares) $f_{\rm CEO}$ is stabilized to 20 MHz in the ERGO comb, which can be realized in principle at two current values $I_{\rm low}$ and $I_{\rm high}$.



Fig. 2. (Color online) Frequency noise power spectral density (PSD) of the CEO beat measured at two different pump currents $I_{\rm low}$ and $I_{\rm high}$ at which $f_{\rm CEO} = 20$ MHz, located on each side of the reversal point. Only the noise that exceeds the β -separation line $S_{\delta\nu}(f) = (8ln2/\pi^2)f$ (dashed line) contributes to the linewidth of the CEO beat [8]. Inset: corresponding CEO-beat showing a higher signal-to-noise ratio at $I_{\rm high}$ than at $I_{\rm low}$.

measured at different pump currents located on both sides of the reversal point. One notices that the response at high frequency (above $\approx 1 \text{ kHz}$) does not depend (both in amplitude and phase) on the operating pump current, whereas in the low-frequency range, a significant difference is observed, especially in the phase of the transfer function, which changes by 180 deg between low and high pump current. This behavior indicates that two different physical phenomena in the gain medium (with a nonlinear refractive index $n_2 =$ $3.3 \cdot 10^{-16} \text{ cm}^2/\text{W}$) are responsible for the tuning of $f_{\rm CEO}$ with the pump current. Self-phase modulation (Kerr effect) very likely dominates the dependence of f_{CEO} to pump current at high modulation frequency. This contribution is instantaneous and has no reason to be affected by a sign reversal with respect to the pump current. At low frequency, however, a different, slower mechanism is responsible for the significant change of the transfer function observed at different pump currents, in particular in terms of phase. With a relatively tight focus in the gain medium (beam radius estimated by the simulation to 20 µm in the tangential plane and 13 µm in the sagittal plane [10]), thermal effect, such as thermal lensing in the gain medium, is believed to be responsible for this change.

Thermal lensing could change the overlap between the pump and the laser beams in the gain medium as a function of the pump current, and this change may lead to a shift of the CEO frequency in either direction. This thermal effect also constitutes the dominant contribution to the static tuning coefficient $\partial f_{\rm CEO}/\partial I_p$. Its change of sign occurring at the reversal point (see Fig. 1) is in good agreement with the 180 deg phase shift observed in the CEO transfer function at low modulation frequency between the two cases $I_p < I_{\rm reversal}$ and $I_p > I_{\rm reversal}$. This 180 deg phase shift is at the origin of the impossibility to phase-stabilize $f_{\rm CEO}$ at the high pump current $I_{\rm high}$, whereas the stabilization is straightforward at $I_{\rm low}$, as will be discussed.

A general rule of thumb in control systems requires a sufficient phase margin to be reached at the unity gain frequency to keep a feedback loop stable [11]. As a



Fig. 3. (Color online) Amplitude (a) and phase (b) of the dynamic response of f_{CEO} to pump current modulation measured at different pump currents: $I_p = I_{\text{low}}$ below the reversal point (green curves), $I_p = I_{\text{high}}$ above the reversal point (red curves) and $I_p = I_{\text{reversal}}$ at the reversal point (orange curves).

consequence, the overall phase shift accumulated throughout the loop has to be kept sufficiently higher than -180 deg to prevent the system for turning into apositive feedback loop, leading to an unstable operation. In the ERGO comb, the CEO-beat stabilization loop involves a digital phase detector (MenloSystems DXD200) to measure the phase fluctuations between the CEO beat and the 20 MHz reference frequency and a servo controller (MenloSystems PIC201) to close the loop. The transfer function of these two components has been measured in amplitude and phase. Combined with the dynamic response of f_{CEO} to the pump current modulation depicted in Fig. 3, the transfer function of the entire stabilization loop has been determined, both in amplitude and phase, for the two pump currents of interest, as displayed in Fig. 4. The contribution from the dynamic response of the CEO beat to the pump current modulation does not exhibit a significant phase shift up to ≈1 kHz at pump currents below the reversal point (Fig. 3), so that a feedback bandwidth of several kilohertz can be achieved with a sufficient phase margin, resulting in a tight phase lock of the CEO beat. On the other side of the reversal point, a large phase shift in the CEO-beat dynamic response is already reached at a very low frequency, which leads to an overall loop phase shift lower than -180 deg already at 1 Hz Fourier frequency. This explains the impossibility to stabilize the CEO beat at an operating current above the reversal point.

In conclusion, the importance of the CEO dynamics in the self-referencing of a DPSSL frequency comb has been demonstrated. The existence of a significant phase shift of ~180 deg in the dynamic response of $f_{\rm CEO}$ for pump current modulation between the low (a few hertz) and the high (kilohertz) frequency ranges prevents the CEO beat to be phase-stabilized at pump currents above the reversal point. On the contrary, the flat CEO phase response obtained at pump currents below the reversal point enables an overall feedback bandwidth of several kilohertz, which straightforwardly results in a tight CEO lock. Our results demonstrate that a high signal-to-noise ratio and a low-noise CEO beat are only prerequisites for



Fig. 4. (Color online) Gain (a) and phase (b) of the overall CEO stabilization loop transfer function at the two different pump currents $I_p = I_{\text{low}}$ (top curves) and $I_p = I_{\text{high}}$ (bottom curves). At $I_p = I_{\text{low}}$, the phase margin is sufficient to achieve a feedback bandwidth (labeled as f_{0dB}) of several kHz. At $I_p = I_{\text{high}}$, the phase is < – 180 degrees already at 1 Hz Fourier frequency, which prevents the CEO stabilization.

the self-referencing of an optical frequency comb, but these conditions may not be sufficient in cases for which the CEO dynamics plays a crucial role. Although this observation was made for a particular DPSSL comb, similar effects might also occur in other types of combs and have to be considered when a new type of ultrafast laser is aimed at being self-referenced.

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