

Inappropriate application of logarithmic transformations for allometric power functions of morphometric data on acanthocephalan worms

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Abstract

Logarithmic transformation is used to transform multiplicative phenomena to a linear, additive scale. Some workers recommend the transformation $x' = \log x + 1$ when observed values are equal to zero. However, when working with morphometric data, this transformation is inappropriate because it arbitrarily changes the nature of the relationship between body parts. To illustrate this point, this contribution re-analyses a data set from the recent literature.

Key words: Acanthocephala, allometry, logarithmic transformation, power function, life history trade-off

The allometric relationship between two body parts (y and x) is often described by the power function where $y = b(x)^a$ (Huxley, 1924; Bookstein, 1978). For graphing and statistical analysis workers often use log–log transformations to convert the above power function to a linear model of the form $\log y = \log b + a(\log x)$. Transforming multiplicative phenomena to a linear, additive scale simultaneously improves homoscedasticity and normality of the data which are two important assumptions in parametric hypothesis testing (Sokal & Rohlf, 1995; Zar, 1999). For these reasons morphometric data are almost always log-transformed.

Some commonly used statistical texts such as Zar (1999), suggest that the $x' = \log(x + 1)$ transformation is preferred on theoretical grounds when the observed values contain zeros, as is commonly the case in count data (Underwood, 1997). However, for morphometric data, adding a constant prior to taking the logarithm is not appropriate because this changes the nature of the allometric relationship in a completely arbitrary manner. Here I use an example from the recent literature to illustrate the error of using this transformation on morphometric data.

Poulin & Morand (2000)¹ examined a trade-off between somatic growth (body size) and male reproductive effort (testis size) across 112 species of acanthocephalan worms. In this group males use copulatory plugs to prevent insemination by other

males. Under the premise of limited sperm competition, the authors predicted that relative testis size would decrease across species as male–male competition intensified. Sexual size dimorphism was used as an index of male–male competition and was measured as the residuals from a regression of male body volume against female body volume. Similarly, relative testis size was measured as the residual after regressing testis volume against male body volume. In each of these allometric regressions the authors used the transformation $x' = \log(x + 1)$ to transform female, male and testis volume.

The authors found that the relationship between testis volume and male body volume was best described by a quadratic equation (Fig. 1(a); compared with fig. 3(a) in Poulin & Morand, 2000). However, this is strictly an artefact of the transformation. If testis and male body volume are expressed in μm^3 rather than mm^3 the relationship becomes linear (Fig. 1(b)). Also note that the eight outliers (marked in black) in Fig. 1(b) are obscured by the transformation in Fig 1(a). Although my estimates of the quadratic regression coefficients were slightly different from the original analysis ($y = -0.149x + 0.150x^2 + 0.064$, compared with $y = -0.154x + 0.152x^2 + 0.067$ in Poulin & Morand, 2000), this does not affect the general conclusions.

What has happened? Testis volume ranged between 0.0002 and 8.294 mm^3 ; a testis volume of 0.2 mm^3 is 1000 times greater than a testis volume of 0.0002 mm^3 . If we add one and then log-transform, the difference between the two testis volumes is $\log 1.2 - \log 1.0002 = 0.07918 - 0.00009 = 0.07909$. However, if we express testis volume in cubic micrometres (200 000 000 and

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¹ Poulin & Morand have seen a draft of this paper.

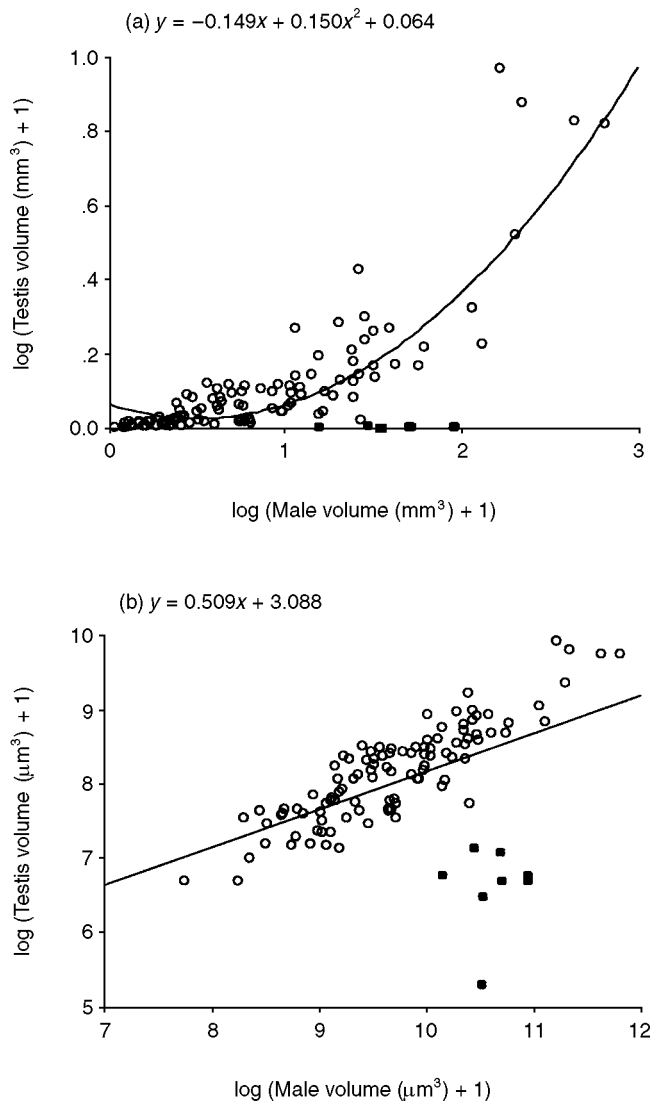


Fig. 1. Relationship between total testis volume and male body volume in parasitic acanthocephalans for species values: (a) in units of mm^3 ; (b) in units of μm^3 . Data are $\log_{10}(x+1)$ -transformed volumes as in the original analysis.

200 000 μm^3 respectively) and repeat this exercise, the difference between the two increases to 3.000.

After using the appropriate transformation ($x' = \log x$), and repeating the analysis across species, male volume correlated more strongly with female volume ($n = 112$, $r^2 = 0.832$, $P < 0.0001$, Fig. 2; compared with $r^2 = 0.794$, see fig. 2(a) in Poulin & Morand, 2000). The relationship is still hypoallometric but the slope is steeper than in the original analysis (slope = 0.880, 95% confidence intervals (CI) = 0.805 to 0.954; compared with slope = 0.767 in Poulin & Morand, 2000).

Similarly, testis volume and male body volume are also correlated (Fig. 3) but not as strongly as in the original analysis ($n = 112$, $r^2 = 0.278$, $P = 0.0000$; compared with $r^2 = 0.640$ in Poulin & Morand, 2000). Again the relationship is hypoallometric (slope = 0.509, 95% CI = 0.354 to 0.663; slope is not compared with Poulin

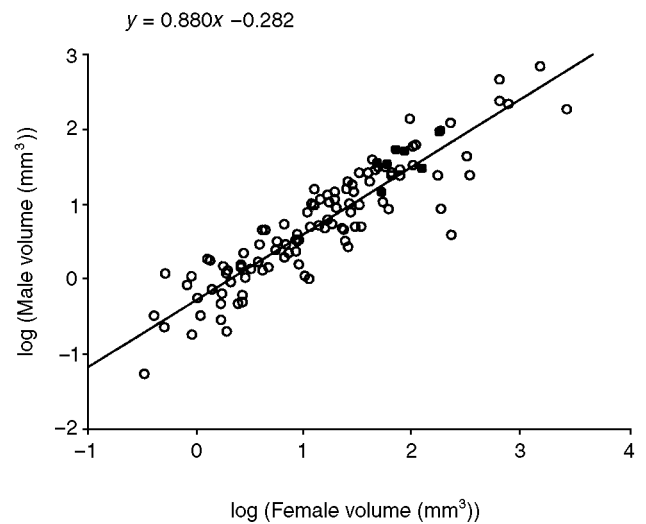


Fig. 2. Relationship between male body volume and female body volume in parasitic acanthocephalans for species values. Data are $\log_{10}(x)$ -transformed volumes.

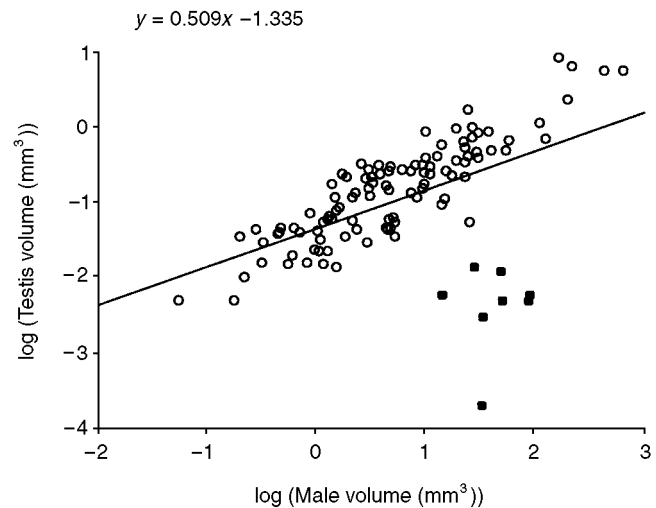


Fig. 3. Relationship between total testis volume and male body volume in parasitic acanthocephalans for species values. Data are $\log_{10}(x)$ -transformed volumes.

& Morand, 2000, because they used a quadratic model) which means that testis volume increases at a slower rate than male volume. If the eight outliers in Fig. 3 are removed, the r^2 value increases to 0.766 and the slope increases to 0.736 (95% CI = 0.656 to 0.816).

Using residuals from the above regressions as indices of male-male competition (male volume corrected for female volume) and male reproductive investment (testis volume corrected for male volume), the negative relationship between relative male size and relative investment in testis growth across species becomes weak (Fig. 4, $n = 112$, $r = -0.173$, $P = 0.069$, compared with $r = -0.242$, $P = 0.0086$ in Poulin & Morand, 2000). However, the relationship remains if residuals are used from the analysis that excludes the eight outliers ($n = 104$, $r = -0.266$, $P = 0.006$). Hence the evidence for an interspecific trade-off between testis and somatic

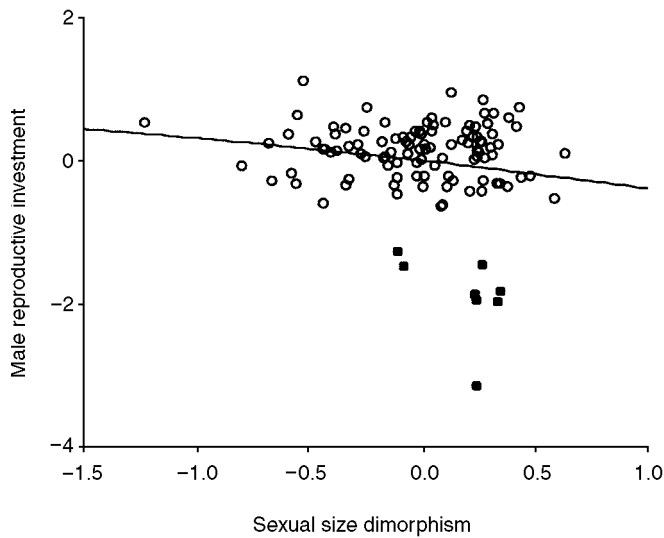


Fig. 4. Relationship between male reproductive investment (testis volume corrected for male body volume) and sexual size dimorphism (male body volume corrected for female body volume) for species values, $r^2=0.0298$. If the residuals are recalculated without including the eight outliers (i.e. $n=104$) the r^2 increases to 0.0706.

growth as male–male competition intensifies is not as clear as Poulin & Morand (2000) suggest.

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