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GTAPINGAMS, version 9: Multiregional and small open economy models with alternative demand systems

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GTAPINGAMS, version 9: Multiregional and small open economy models with alternative demand systems*

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Abstract

This paper describes the implementation in GAMS of an economic equilibrium model based on the GTAP version 9 dataset. We call this model and the ancillary programming tools GTAPINGAMS, version 9. Relative to previous installments of GTAPINGAMS, an innovation in this model is that it can easily switch between global multiregional (GMR) and small open economy (SOE) closures. We also include the possibility to evaluate results for alternatives representations of final demand, based on Cobb-Douglas preferences, linear expenditure system or the constant difference in elasticities function. In this paper we outline the model structure, document the associated equilibrium conditions and describe computer programs which calibrate the model to the desired regional and sectoral aggregation from the GTAP9 dataset. We perform a few calculations which illustrate how alternative structural assumptions influence the policy conclusions derived from the model.

Keywords: Applied economic analysis; Multiregional models; Small open economy models; Regulation; Trade policy; Computational models; Calibration.

JEL Codes: C6, C8, D5, F1, R1.

*This paper borrows material from the unpublished manuscripts Rutherford (1997) and Rutherford (2005). The computer code associated with this paper can be accessed via http://www.mpsge.org/gtap9ingams.zip. We would like to thank Christoph Boehringer, Justin Caron and Jan Imhof for their contributions to this work. We remain responsible for any remaining error.

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1 Introduction

Analysis of policies affecting markets in multiple countries requires both data and theory. The GTAP consortium provides data, and the analysts confronts this data with a theoretical perspective.[1] Despite some limitations in data coverage and quality, a key practical constraint lies in the informed translation of theoretical insights into quantitative policy evidence. Our paper is intended to facilitate this process. We provide computational tools to exploit GTAP data in conjunction with general equilibrium theory, and thereby contribute to the development of computable general equilibrium (CGE) analysis. In a nutshell, CGE models rationalize micro-consistent input-output matrix with a standard Arrow-Debreu general equilibrium representation of the economy,[2] and quantify *ex-ante* the impact of a policy relative to an observed state of affair.

While CGE models represent an increasingly important area of policy research, quantitative results from such analysis are inherently model-dependent, which offers a challenge for their role to inform policy-makers. Therefore, expanding the set of modeling strategies is important both to further the academic state-of-the-art and to foster confidence in the use of the results for policy-design purposes. With this in mind, this paper introduces a new version of the GTAPINGAMS model, building on previous contributions by Rutherford (1997) and Rutherford (2005). As we detail below, this version of the distribution includes both *global multi-regional* (GMR) and *small open economy* (SOE) versions of the GTAPINGAMS model, in which the SOE model may be single or multi-regional. In addition, users can now toggle across several representations of final consumer demand, and thus test robustness of model results in that dimension.

The version of the model we present is based on version 9 of the GTAP database (Aguiar

[1] The Global Trade Analysis Project (GTAP) is a research program initiated in 1992 to provide the economic research community with a global economic dataset for use in the quantitative analysis of international economic issues. GTAP has lead to the establishment of a global network of researchers who share a common interest of multi-region trade analysis and related issues, notably climate and energy policy. The GTAP research is coordinated by Thomas Hertel, Director of the Center for Global Trade Analysis at Purdue University (see notably Hertel, 1997). As Deputy Director of this Center, Robert McDougall oversees the data base work (see e.g. McDougall, 2005). Software development within the GTAP project has been assisted greatly by the efforts of Ken Pearson, Mark Horridge and other researchers from Centre of Policy Studies, Monash University (see http://www.gtap.org for a list of applications based on the GTAP framework).

[2] Input-output matrices provide data on value flows between economic sectors and regions for primary production factors, intermediate goods and final consumption products. In their micro-consistent versions, such as those provided in the GTAP data, these matrices provide a complete representation of the economy, such that no value is lost in transaction. The ensuing dataset represents value flows in a closed economic system.

et al., 2016), which represents global production and trade for 140 country/regions, 57 commodities and nine primary factors, two of which (*land* and *natural resources*) are "sluggish" (imperfectly mobile across sectors); see Appendix A for a complete list of sectors and factors of production. The data characterize intermediate demand and bilateral trade in three alternative base years (2004, 2007 and 2011), including tax rates on imports and exports and other indirect taxes. Our implementation gives users the option to exploit recently developed tools for parallel computation, which can significantly increase computational speed and makes it less cumbersome to exploit the full dimensionality of the dataset. The dataset also provides trade elasticities and Armington elasticities, which can be used to control the international trade response of the model, as well as income and price elasticities for final demand to parametrize final demand. See Narayanan (2015) for a comprehensive description of key features of the GTAP 9 database.

By design, the GTAP database is well suited for the formulation of quantitative economic models, which in turn can be used to simulate the effect of policies. The principal programming language for GTAP data and modeling work is GEMPACK (Harrison and Pearson, 1996, 2007). In the GEMPACK framework the model is solved as a system of nonlinear equations. The present paper describes a version of the GTAP model which has been implemented in GAMS. The GAMS model is essentially implemented as a nonlinear system of equations, although it can be posed either as a (primal) constrained non-linear optimization problem or a mixed complementarity problem (MCP).[3] One potentially important difference between the GEMPACK framework and previous GTAPINGAMS releases is the representation of final demand:[4] GEMPACK employs a constant difference of elasticities (CDE) demand system (Hanoch, 1975), which allows introducing evidence about own-price and income elasticities, while GTAPINGAMS models uses Cobb-Douglas preferences to represent final demand. The present distribution includes an implementation of the CDE demand system together with a least squares calibration code for the CDE which works with the full GTAP dataset and many aggregations. In addition to the original Cobb-Douglas representation, we also provide the

[3] The canonical GTAPINGAMS model is essentially a nonlinear system of equations, as the model does not include activity analysis nor does it rely on free disposal. Formulation as an MCP imposes some modeling discipline, as equations must be explicitly linked with variables. Extensions of the model to incorporate, e.g., tariff quotas (Hertel et al., 2009) or quantitative restrictions on carbon emissions which may or may not be binding (Böhringer et al., 2016).

[4] See Rutherford (2005) for a discussion of the key differences between the two modeling frameworks.

option to use a linear expenditure system (LES) representation, with code to parametrize the function to match own-price and income elasticity data provided in the GTAP dataset.

Therefore, the first and principal contribution of this paper is to document programs included in the GTAPINGAMS version 9 package. These programs permit analyst to: (i) select an appropriate level of sectoral and regional aggregation from the GTAP 9 dataset and hence target a representation that is relevant to the analysis of interest; (ii) systematically filter out small and economically insignificant activity accounts, which significantly improves robustness and efficiency of numerically solution; and (iii) re-balance the filtered dataset to ensure that it represents a micro-consistent system and remains consistent with estimates of own-price and income elasticities of demand. The paper also provides details about core CGE models, which are calibrated to the full GTAP datasets for 2004, 2007 and 2011 benchmark years.

A second contribution is to introduce a new SOE version of the model, and to propose an approach to calibrate trade elasticity parameters in the SOE model to match the trade response of the GMR model. This makes it possible to produce consistent comparisons across trade closures. The SOE formulation necessitates a modest generalization of the standard GTAP model in which goods produced for domestic and export markets are perfect substitutes. Following a large literature based on Devarajan et al. (1990), our SOE model differentiates goods produced for domestic and export markets, applying a constant-elasticity-of-transformation (CET) revenue function. The SOE model can be used independently of the GMR model for analysis of trade issues in a one or more countries with fixed terms of trade relative to the rest of the world. It may also be used in combination with the multiregional model to decompose the contribution of changes in terms of trade to economic outcomes, as in Böhringer and Rutherford (2002).

The models developed here offer a very rich framework to evaluate the international impacts of policies. In the present paper, however, we limit ourselves to a simple illustration of how the GMR and SOE models can be used to better understand the role of terms of trade in policy experiments. More specifically, in these simulations we assess the economic impact of *unilateral, proportional* adjustment of regional import tariffs and export taxes in the US and China. In the GTAP database, average trade taxes are a little less than 2% in the US, and over 4% in China. Our results suggest that the optimal rate depends crucially on whether one works with a multiregional or open economy closure, whereas the representation of final demand only has a minor impact on the quantitative results.

The remaining of this paper is structured as follows. Section 2 introduces the core GTAPINGAMS model. Section 3 presents the equilibrium conditions associated with the dual (cost minimization) version of the model, which form the basis for the GAMS software. Section 4 introduces the computer code for the model. It further discusses issues about data aggregation, filtering, and re-balancing, and how GMR and SOE models can be calibrated to obtain empirically consistent trade responses. Section 5 reports some forensic calculations of our illustrative policy example. Concluding comments are provided in Section 6. The paper is complemented by an online appendix which provides all the GAMS files discussed in the paper (see http://www.mpsge.org/gtap9ingams.zip).[5]

2 Canonical Models

This section introduces the structure of core GMR and SOE models included in the GTAPINGAMS distribution. Both models are static, and track the production and distribution of goods in the global economy. In GTAP the world is divided into 140 regions (typically representing individual countries) and 57 commodities (or goods), but computational constraints tend to limit the number of regions and goods which can be included in a single model.[6] In each region, final demand structure is composed of public and private expenditure across goods. Decisions about the allocation of resources are decentralized, and the representation of behavior by consumers and firms in the model follows the canonical microeconomic optimization framework: (i) consumers maximize welfare subject to budget constraint with fixed levels of investment and public output; (ii) producers combine intermediate inputs, and primary factors (several categories of labor, land, resources and physical capital) at least cost for given technology. Details about GTAP sectors and primary factors are provided in Appendix A.

[5] The computer programs described here require data files from the GTAP 9 Flexagg package, `flexagg9aY04.zip`, `flexagg9aY07.zip` and `flexagg9aY11.zip`. License for these data files must be purchased separately by the users.

[6] Datasets are easily aggregated which can be useful for both empirical applications focusing on a subset of regions and for debugging steps at the model formulation stage.

While GMR and SOE models are based on the same data and employ the same representation of production technology and consumer preferences, they of course differ in their representation of trade. The GMR model is based on a standard Armington representation of bilateral trade flows. An implication is that production and consumption decisions in a given country (or group of countries) will affect world prices, and the magnitude of this effect will mainly depend on the elasticity estimates (a measure of country-level market-power). By contrast, the SOE model considers the case in which production and consumption decision in a country (or group of country) do not affect world prices. This is modeled by treating supply and demand by the "rest of the world" (ROW, i.e. countries that are not considered relevant for the SOE analysis) as perfectly elastic. Corner solutions are avoided through the assumption that output destined for the domestic and export markets are differentiated products (see De Melo and Robinson, 1989).[7] Therefore, from the perspective of a given country or group of countries, the impact of policy evaluated in GMR and SOE models will differ through their impact on the terms of trade.

In the following, we start by describing the basic notation, and then present the structure of the data together with benchmark accounting identities. We then present a "primal" description of agents' optimization problems (i.e. specified in terms of quantity variables), which leads to the equilibrium conditions presented in the subsequent section.

2.1 Notation

The notation used in the model is summarized in the Tables 1 - 3. Table 1 defines the various dimensions which characterize an instance of the model, including the set of sectors/commodities (i, j), the set of regions (r, s), the set of factors of production (f). Set g combines the production sectors i and private and public consumption demand (indices `"C"` and `"G"`) and investment demand (index `"I"`). It allows for a much tighter formulation of the model as they can all be conceived of "goods" produced in similar fashion. To simplify the exposition of the model, however, we describe private consumption, public consumption and investment demand as stand alone components.

[7] In the present implementation, international closure between the region of interest and the ROW is achieved by fixing the value of the current account at its benchmark level, and permitting the real exchange rate to clear the market. Other assumptions are of course possible and are left for future research.

Table 1: Definitions of set indices

Set	Definition
i, j	Sectors, an aggregation of the 57 sectors in the GTAP 9 database
g	Production sectors i, plus private consumption "C", public demand "G" and investment "I"
r, s	Regions, an aggregation of the 140 regions in the GTAP 9 database
f	Factors of production (consisting of *mobile factors*, $f \in m_f$, four categories of skilled labor (i. officials, mangers and legislators (ISCO-88 Major Groups 1-2), ii. technicians and associated professionals, iii. clerks, and iv. service and market sales workers), unskilled labor, capital, and *sector-specific*, $f \in s_f$, agricultural land and other resources)

Table 2: Definitions of activity levels (quantity variables)

Variable	Definition	GAMS variable	Benchmark (GTAP) value
Y_{ir}	Production	`Y(i,r)`	`vom(i,r)`
C_r	Discretionary consumption	`Y("c",r)`	`vom("c",r)`
G_r	Aggregate public	`Y("g",r)`	`vom("g",r)`
I_r	Aggregate investment	`Y("i",r)`	`vom("i",r)`
M_{ir}	Aggregate imports	`M(i,r)`	`vim(i,r)`
X_{ir}	Trade flows to or from rest of world regions	`X(i,r)`	`vem(i,r)`
FT_{fr}	Factor transformation	`FT(f,r)`	`evom(f,r)`
YT_j	International transport services	`YT(j)`	`vtw(j)`

Table 2 defines the primal variables (activity levels) which characterize an equilibrium. The model determines values of all the variables except international capital flows, a parameter which would be determined endogenously in an intertemporal model. Table 2 also displays the concordance between the variables and their GAMS equivalents.

Table 3 defines the relative price variables for goods and factors in the model. As is the case in any Shoven-Whalley CGE model, the equilibrium conditions determine *relative* rather than *nominal* prices.

While the core GTAPINGAMS model is base on Cobb-Douglas final demand, the canonical

Table 3: Definition of price variables

Variable	Definition	GAMS variable
p_r^C	Final demand price index for the Cobb-Douglas demand system	`P("C",r)`
p_r^G	Public provision price index	`P("G",r)`
p_r^I	Investment price index	`P("I",r)`
p_{ir}^Y	Supply price, gross of indirect producer taxes[8]	`P(i,r)`
p_{ir}^M	Import price, gross of export taxes and tariffs.	`PM(i,r)`
p_{ir}^E	Export price from endogenous or exogenous regions.	`PE(i,r)`
p_j^T	Marginal cost of transport services	`PT(j)`
$p_{f,r}^F$	Factor prices for factors (mobile factors m_f include labor, land and resources)	`PF(f,j)`
$p_{s_f,ir}^S$	Price of sector-specific primary factors	`PF(sf,i,j)`

Table 4: Additional variables for LES and CDE demand systems

Variable	Definition	GAMS variable
SD_r	Subsistence demand	`Y("sd",r)`
DD_r	Discretionary demand	`Y("dd",r)`
p_r^{DD}	Discretionary demand price index	`P("dd",r)`
p_r^{SD}	Subsistence demand price index	`P("sd",r)`
p_{iCr}^A	Armington composite price representing the market price of goods entering CDE demand.	`PA(i,r)`

models incorporate logic for two additional demand systems: the LES and the CDE demand system. In particular, the LES representation distinguishes between subsistence demand and discretionary demand, and we define the additional variables in Table 4. Similarly, we define an Armington composite price index for products entering CDE final demand.

Finally, Table 5 reports the definition of tax and subsidy rates applied in the model, both in terms of the notation employed to describe the model and that used in the GAMS code.

Table 5: Tax and subsidy rates (net basis unless noted)

Parameter	Definition		GAMS Parameter
t^o_{ir}	Output taxes (gross basis)		`rto(i,r)`
t^f_{fir}	Factor taxes		`rtf(f,i,r)`
t^{fd}_{ijr}	Intermediate input taxes	Domestic	`rtfd(i,j,r)`
t^{fi}_{ijr}		Imported	`rtfi(i,j,r)`
t^{pd}_{ir}	Consumption taxes[9]	Domestic	`rtfd(i,"C",r)`
t^{pi}_{ir}		Imported	`rtfi(i,"C",r)`
t^{gd}_{ir}	Public demand taxes	Domestic	`rtfd(i,"G",r)`
t^{gi}_{ir}		Imported	`rtfi(i,"G",r)`
t^{gd}_{ir}	Investment demand taxes	Domestic	`rtfd(i,"I",r)`
t^{gi}_{ir}		Imported	`rtfi(i,"I",r)`
t^{xs}_{isr}	Export subsidies		`rtxs(i,s,r)`
t^{ms}_{isr}	Import tariffs		`rtms(i,s,r)`

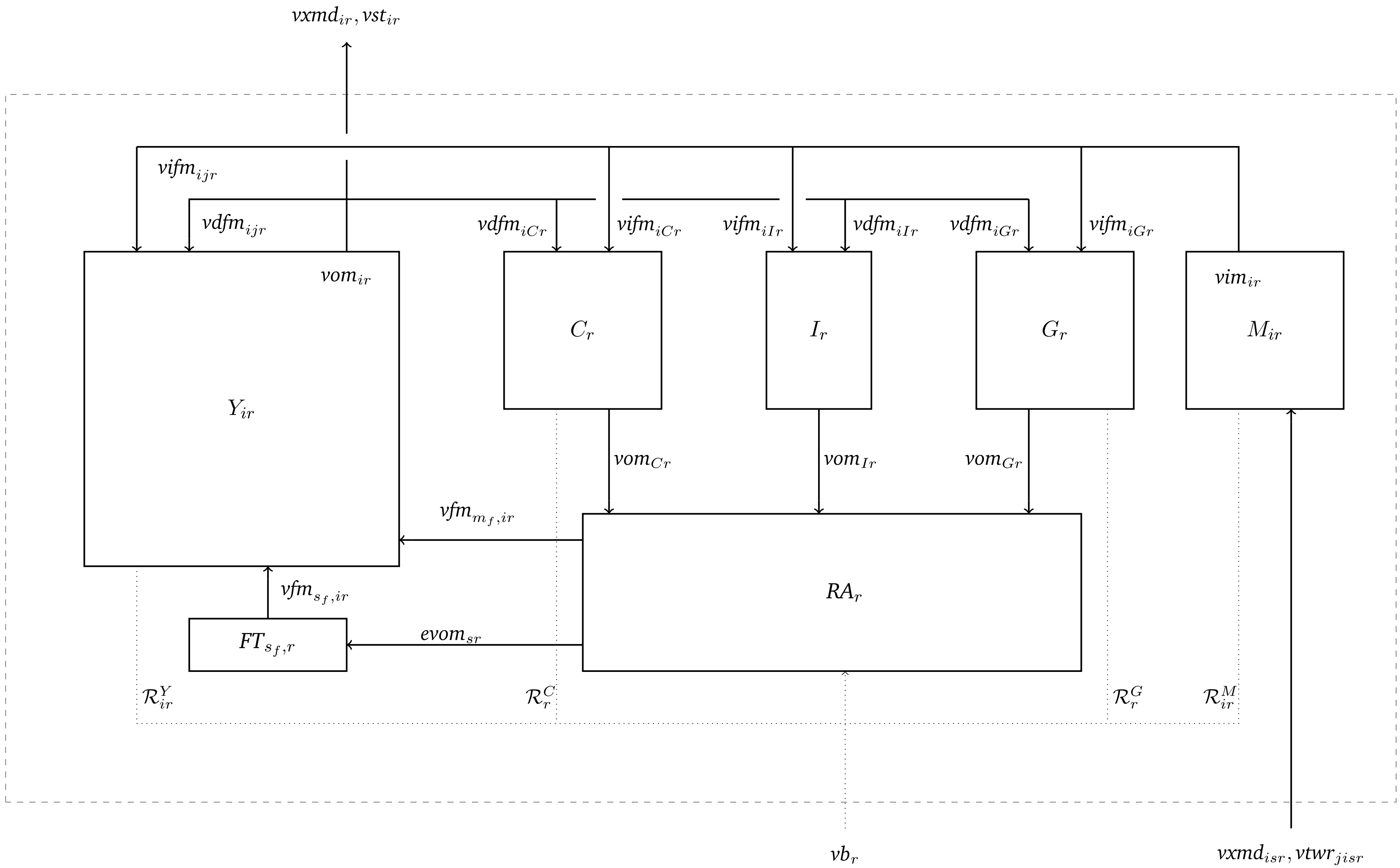
Note that revenues from taxes and subsidy expenditures do not appear as explicit variables in the GTAP database and are defined on the basis of expenditures and tax rates. We come back to this below.

2.2 Benchmark data structure and accounting identities

The economic structure underlying the GTAP dataset and model is illustrated in Figure 1. Symbols in this flow chart correspond to variables in the economic model (see Table 2): Y_{ir} is the production of good i in region r, C_r, I_r and G_r portray private consumption, investment and public demand, respectively, M_{ir} portrays the import of good i into region r, RA_r stand for representative consumers, and $FT_{s_f r}$ is the activity through which the set of sector-specific factors of production (s_f) are allocated to individual sectors. Further, solid lines represent commodity and factor market flows, while dotted lines indicate tax revenues and transfers.

Domestic and imported goods markets are represented by horizontal lines at the top of the figure. Domestic production (vom_{ir}) is distributed to exports ($vxmd_{irs}$), international transportation services (vst_{ir}), intermediate demand ($vdfm_{ijr}$), household consumption ($vdfm_{iCr}$),

Figure 1: Regional economic structure



investment ($vdfm_{iIr}$), and government consumption ($vdfm_{iGr}$).[10] The accounting identity in the GTAP 9 dataset is thus:

$$vom_{ir} = \sum_s vxmd_{irs} + vst_{ir} + \sum_j vdfm_{ijr} + vdfm_{iCr} + vdfm_{iIr} + vdfm_{iGr} \,,$$

where j indexes all goods. Similarly, imported goods (with aggregate value vim_{ir}) enter intermediate demand ($vifm_{ijr}$), private consumption ($vifm_{iCr} + vifm_{iSDr}$) and public consumption ($vifm_{iGr}$). The accounting identity for these flows is thus:

$$vim_{ir} = \sum_j vifm_{ijr} + vifm_{iCr} + vifm_{iSDr} + vifm_{iGr} \,.$$

Inputs to production of good i (Y_{ir}) include intermediate inputs (domestic $vdfm_{ijr}$ and imported $vifm_{ijr}$), mobile factors of production ($vfm_{m_f,ir}$, where m_f is a subset of the set f designating all factors of production), and sector-specific factors of production ($vfm_{s_f,ir}$, $s_f \subset f$). Factor market equilibrium is given by an identity relating the value of factor payments to factor income:

$$\sum_i vfm_{fir} = evom_{fr} \,,$$

and factor earnings accrue to households.

International market clearance conditions require that region r exports of good i ($vxmd_{ir}$ at the top of the figure) equal the imports of the same good from the same region summed across all trading partners ($vxmd_{isr}$ at the bottom of the figure):

$$vxm_{ir} = \sum_s vxmd_{irs} \,,$$

where s, an alias for r, indexes regions. Likewise, market clearance conditions apply for international transportation services. The supply-demand balance in the market for transportation service j requires that the sum across all regions of service exports (vst_{ir}, at the top of the figure) equals the sum across all bilateral trade flows of service inputs ($vtwr_{jisr}$ at the

[10] Recall that in the GAMS implementation of the model the index `"g"` includes all sectors represented in the model plus private consumption `"C"`, public demand `"G"` and investment `"I"`. See Table 1. For the LES demand representation, it also includes price indices for discretionary demand `"dd"` and subsistence demand `"sd"`.

bottom of the figure):

$$\sum_r vst_{jr} = \sum_{isr} vtwr_{jisr}$$

Turning to tax revenues and transfers, shown as dotted lines in figure 1, flows labeled with $\mathcal{R}$ correspond to tax revenues. For each country, tax flows consist of indirect taxes on production/exports of each good ($\mathcal{R}^Y_{ir}$), on consumption ($\mathcal{R}^C_r$), on public demand ($\mathcal{R}^G_r$) and on imports ($\mathcal{R}^M_{ir}$). The regional budget constraint thus relates tax payments ($\mathcal{R}^Y_{ir}, \mathcal{R}^C_r, \mathcal{R}^G_r, \mathcal{R}^M_{ir}$), factor income ($evom_{fr}$), and the current account deficit (i.e., net transfers from abroad, vb_r) to total private consumption expenditure vom_{Cr}, total public consumption expenditure vom_{Gr}, and total investment vom_{Ir}, yielding:

$$vom_{Cr} + vom_{Gr} + vom_{Ir} = \sum_f evom_{fr} + \sum_i \mathcal{R}^Y_{ir} + \mathcal{R}^C_r + \mathcal{R}^G_r + \sum_i \mathcal{R}^M_{ir} + vb_r \, .$$

To this point we have outlined two types of consistency conditions which are part of the GTAP database: market clearance (supply = demand for all goods and factors), and income balance (net income = net expenditure). A third set of identities involve net operating profits by all sectors in the economy. In the core GTAP model "production" takes place under conditions of perfect competition with constant returns to scale, hence there are no excess profits, and the cost of inputs must equal the value of outputs. This condition applies for each production sector:

$$\begin{aligned}
Y_{ir}&\colon\ \textstyle\sum_f vfm_{fir} + \sum_j \left(vdfm_{ijr} + vifm_{ijr}\right) + \mathcal{R}^Y_{ir} = vom_{ir}\ ,\\
C_r&\colon\ \textstyle\sum_i \left(vdfm_{iCr} + vifm_{iCr}\right) + \mathcal{R}^C_{ir} = vom_{Cr}\ ,\\
I_r&\colon\ \textstyle\sum_i vdfm_{iIr} = vom_{Ir}\\
G_r&\colon\ \textstyle\sum_i \left(vdfm_{iGr} + vifm_{iGr}\right) + \mathcal{R}^G_{ir} = vom_{Gr}\ ,\\
M_{ir}&\colon\ \textstyle\sum_s \left(vxmd_{isr} + \sum_j vtwr_{jisr}\right) + \mathcal{R}^M_{ir} = vim_{ir}\\
FT_{fr}&\colon\ evom_{fr} = \textstyle\sum_i vfm_{fir}
\end{aligned}$$

2.3 Decentralized optimization problems

The benchmark identities presented in the previous section indicate the market clearance, zero profit and income balance conditions which define the GTAP model. The displayed equations do not, however, characterize the behavior of agents in the model. In a competitive equilibrium setting, the standard assumption of optimizing atomistic agents applies for both producers and consumers. This section lays out the optimization problem of each component in the model, and thereby provides the structure of production technology (production functions) and preferences (characterizing final demand), as well as the representation of trade. We also highlight where conceptual differences between GMR and SOE models intervene.

Note that in order to simplify notation, we denote decision variables corresponding to the benchmark data structures with the initial "v" replaced by "d." Hence, while $vdfm_{jir}$ represents benchmark data on intermediate demand for good j in the production of good i in region r, $ddfm_{jir}$ represents the corresponding decision variable in the equilibrium model. This approach to the scaling of variables is consistent with the GAMS code, and it provides a flexible and transparent approach with respect to the calibration of activity variables.

2.3.1 Production technology

Starting with producers, profit maximization in the constant returns to scale setting is equivalent to cost minimization subject to technical constraints. For sector Y_{ir} we characterize input choices as though they arose from minimization of unit costs:

$$
\begin{aligned}
\min_{\mathit{ddfm},\mathit{dfm},\mathit{difm}} \quad & c^D_{ir} + c^M_{ir} + c^F_{ir} \\
s.t. \quad & c^D_{ir} = \sum_j p^Y_{jr}(1+t^{fd}_{jir})\mathit{ddfm}_{jir} \\
& c^M_{ir} = \sum_j p^M_{jr}(1+t^{fi}_{jir})\mathit{difm}_{jir} \\
& c^F_{ir} = \sum_f (p^F_{fr}|_{f\in m_f} + p^S_{fir}|_{f\in s})(1+t^f_{fir})\mathit{dfm}_{fir} \\
& F_{ir}(\mathit{ddfm},\mathit{difm},\mathit{dfm}) = Y_{ir}
\end{aligned}
\tag{1}
$$

where $F(\cdot)$ represents the production function, which is described by a nested constant-elasticity-of-substitution (CES) form, with structure displayed in Figure 2.

In the figure, σ values at different nests represent substitution elasticities between inputs, with $\sigma^D_i = \mathit{esubd}_i$ measuring substitution possibility between intermediate inputs produced domestically and imported from abroad, and esubva_i representing substitution possibilities between primary inputs in the value added nest. Both parameters are provided in the GTAP 9 database (see Hertel and van der Mensbrugghe, 2015). Note further that the specific source of tax revenue is indicated in this figure, consisting of output taxes, taxes on intermediate inputs and taxes on factor demands, all of which are applied on an ad-valorem basis.

Figure 2: CES nesting structure for production function $Y_{ir} = F_{ir}(\mathit{ddfm},\mathit{difm},\mathit{dfm})$

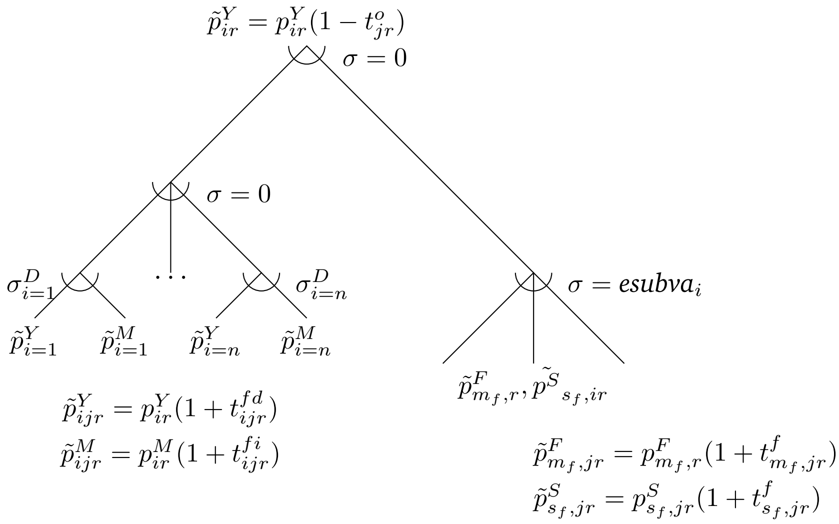
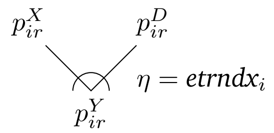


Figure 3: CET transformation between domestic and export markets $Y_{ir} = G_{IR}(\mathit{dxm}, \mathit{ddm})$



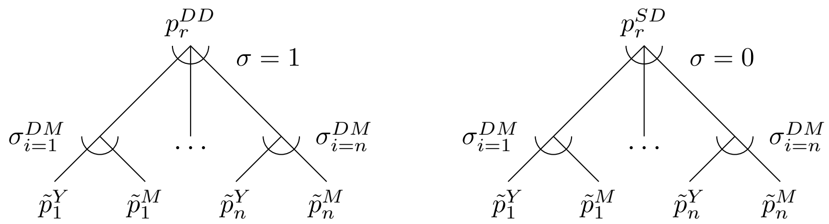
One important difference between GMR and SOE models occurs in the differentiation of output for domestic and export markets. In particular, the supply of goods to domestic and export markets are portrayed as arising from the following profit-maximization problem:

$$\max_{\mathit{dxm},\mathit{ddm}} \quad p^D_{ir}\mathit{ddm}_{ir} + p^X_{ir}\mathit{dxm}_{ir} \tag{2}$$

$$s.t. \quad G_{ir}(\mathit{ddm}_{ir}, \mathit{dxm}_{ir}) = Y_{ir} \tag{3}$$

where G is the CET function with structure illustrated in Figure 3. In the GMR model, transformation elasticities $\eta^{DX}_{ir} = \mathit{etrndx}_i$ are set to infinity, which is the default value in the GTAP 9 database. This implies that the supply price of output p^Y_{jr} is the same if output is used locally or in a different region. By contrast, in the SOE model we have that $\mathit{etrndx}_i < \infty$, which implies that output price may differ when supplied domestically or abroad. In this case, $p^Y jr$ in equation (1) is replaced by p^D_{jr}. We come back to the choice of etrndx_i in the SOE model when we present our illustrative calculations.

Figure 4: LES nesting for discretionary and subsistence consumption



$$\tilde{p}_{ir}^{Y} = p_{ir}^{Y}(1 + t_{ir}^{pd})$$
$$\tilde{p}_{ir}^{M} = p_{ir}^{M}(1 + t_{ir}^{pi})$$

2.3.2 Preferences and final demand

Private consumption consistent with utility maximization is portrayed by minimization of the cost of a given level of aggregate consumption:

$$\min_{ddfm_{iCr}, difm_{iCr}} \quad \sum_i p_{ir}^{Y}(1 + t_{ir}^{pd}) ddfm_{iCr} + p_{ir}^{M}(1 + t_{ir}^{pi}) difm_{iCr} \tag{4}$$
$$s.t. \quad H_r(ddfm_{iCr}, difm_{iCr}) = C_{ir}$$

where H_r represents final demand from the representative consumer.

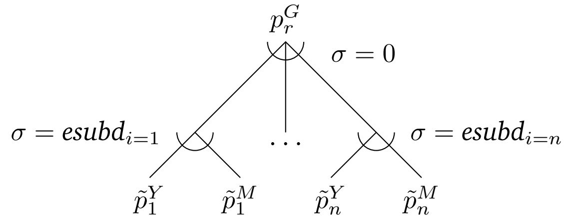
Final demand in the core model is characterized by Cobb-Douglas preferences. Alternative specifications included in the model logic include both the LES and CDE expenditure.[11] The nested discretionary and subsistence demand functions are is displayed in Figure 4.

2.3.3 Government and public consumption

Public consumption in the model is represented as a fixed coefficient (Leontief) aggregation of domestic-import composites. This formulation introduces substitution at the second level between domestic and imported inputs while holding sectoral commodity aggregates constant. Figure 5 illustrates the functional form.

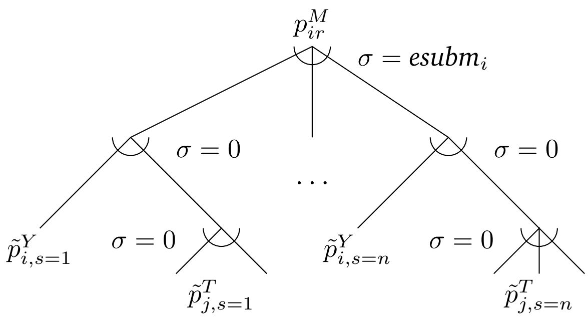
[11] The LES model involves subsistence and discretionary expenditure, which are represented through composite price indices representing CES aggregates of both domestic and imported inputs, `P("DD",r)` and `P("SD",r)`, respectively. The CDE model requires the introduction of trade aggregation activities (`A(i,r)`) and associated Armington composite price indices, `PA(i,r)`.

Figure 5: Nesting structure for public consumption $G_r = G_r(\mathit{ddfm}_{iGr}, \mathit{difm}_{iGr})$



$$\tilde{p}_{ir}^Y = p_{ir}^Y(1 + t_{ir}^{gd})$$
$$\tilde{p}_{ir}^M = p_{ir}^M(1 + t_{ir}^{gi})$$

Figure 6: Armington aggregation of traded goods $M_{ir} = A_{ir}(\mathit{dxmd,dtwr})$



$$\tilde{p}_{is}^Y = p_{is}^Y(1 - t_{isr}^{xs})(1 + t_{isr}^{ms})$$
$$\tilde{p}_{js}^T = p_j^T(1 + t_{isr}^{ms})$$

2.3.4 International trade

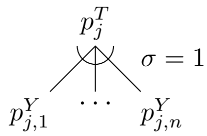
The choice among imports from different trading partners is based on Armington's idea of regionally differentiated products. The following cost minimization problem formalizes this choice:

$$\min_{\mathit{dxmd,dtwr}} \quad \sum_s (1 + t_{isr}^{ms}) \left(p_{is}^Y (1 - t_{isr}^{xs}) \mathit{dxmd}_{isr} + \sum_j p_j^T \mathit{dtwr}_{jisr} \right) \qquad (5)$$
$$s.t. \quad A_{ir}(\mathit{dxmd,dtwr}) = M_{ir}$$

where A is the import aggregation function, described by the nested CES-Leontief function shown in Figure 6. In the case of the SOE model, in which $\mathit{etrndx}_i < \infty$, p_{is}^Y in (5) is replaced by p_{is}^X.

Note that transportation services enter on a proportional basis with imports from different countries, reflecting differences in unit transportation margins across different goods

Figure 7: International transportation services aggregator $YT_j = T_j(dst)$



and trading partners. Therefore, substitution at the top level in an Armington composite involves trading off of both imported goods and associated transportation services. Trade flows are subject to export subsidies and import tariffs, with subsidies paid by government in the exporting region, and tariffs collected by government in the importing region.

The provision of international transportation services is modeled through an aggregation of transportation services exported from countries throughout the world. More specifically, we consider the following cost minimization problem for the aggregation of transportation services:

$$\min_{dst} \quad \sum_r p_{ir}^Y dst_{ir} \quad \text{s.t.} \quad T_i(dst) = YT_i$$

where the aggregation function T_i combines transport service exports from multiple regions. The functional form which aggregates services from different regions is illustrated in Figure 7.

2.3.5 Supply of sector-specific factors

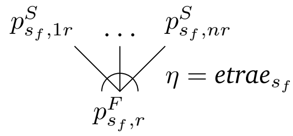
Land and natural resources are portrayed as sector-specific factors of production supplied through constant-elasticity-of-transformation (CET) production function allocates composite factors to sectoral markets. Formally, the supply of sectoral factors of production is modeled through the following profit-maximization problem:

$$\max_{dfm} \qquad \sum_j dfm_{s_f,jr} p_{s_f,jr}^S \tag{6}$$

$$s.t. \quad \Gamma_{s_f,r}(dfm) = evom_{s_f,r} \tag{7}$$

where Γ is the CET function with structure illustrated in Figure 8. Note that in the figure η represents transformation elasticities provided in the GTAP 9 database.

Figure 8: Sector-specific factor CET transformation function $FT_{s_f} = \Gamma_{s_f}(dfm)$



3 Model formulation through equilibrium conditions

An Arrow-Debreu model concerns itself with the interactions of decentralized decisions by consumers and producers in markets. Mathiesen (1985) proposed a representation of this class of models in which two types of equations define an equilibrium: zero profit and market clearance. The corresponding variables defining an equilibrium are activity levels (for constant-returns-to-scale firms) and commodity prices.[12] Here we extend Mathiesen's framework with a third class of variables corresponding to consumer income levels. Commodity markets encompass primary endowments of households with producer outputs. In equilibrium the aggregate supply of each good must be at least as great as total intermediate and final demand. Initial endowments are exogenous. Producer supplies and demands are defined by producer activity levels and relative prices. Final demands are determined by market prices.

Economists who have worked with conventional textbook equilibrium models can find Mathiesen's framework to be somewhat opaque because many quantity variables need not be explicitly specified in the model. Variables such as final demand by consumers, factor demands by producers and commodity supplies by producers, are defined implicitly in Mathiesen's model. For example, given equilibrium prices for primary factors, consumer incomes can be computed, and given income and goods prices, consumers' demands can then be determined. The consumer demand functions are written down in order to define an equilibrium, but quantities demanded need not appear in the model as separate variables. The same is true of inputs or outputs from the production process: relative prices determine conditional demand, and conditional demand times the activity level represents market demand. Omit-

[12] Under a maintained assumption of perfect competition, Mathiesen (1985) may characterize technology as constant-returns-to-scale without loss of generality. Specifically, decreasing returns are accommodated through introduction of a specific factor, while increasing returns are inconsistent with the assumption of perfect competition. Note that in this environment zero excess profit is consistent with free entry for atomistic firms producing an identical product.

ting decisions variables and suppressing definitional equations corresponding to intermediate and final demand provides significant computational advantages at the cost of a somewhat more complicated model statement.

In the following, we detail (i) zero profit conditions, (ii) market clearance conditions, and (iii) income balance conditions, which in the present case is equivalent to the regional budget constraint. These three sets of conditions form the basic system of equation to be solved. Note that the actual code for the model is implemented both in the algebraic mixed complementarity format (GAMS/MCP, see Rutherford, 1995) and through the more compact formulation afforded by the GAMS/MPSGE syntax (Rutherford, 1999).

3.1 Zero profit (arbitrage) conditions

All production activities in the model are represented by constant-returns-to-scale technologies, and markets are assumed to operate competitively with free entry and exit. As a consequence, equilibrium profits are driven to zero and the price of output reflects the cost of inputs. The following sets of equations relating output price to marginal cost are part of the definition of an equilibrium.[13]

The calculation of unit cost and unit revenue functions involves the definition of a number of ancillary variables (that do not appear in the GAMS code as explicit choice variables). In the following we define ancillary variables in un-numbered equations, indicating that these variables are "optional" in the sense that they may be substituted out of the non-linear system of equations. Moreover, we use the symbol θ to portray value shares from the base year data. In many cases subscripts on these value shares are omitted in order to economize on notation. Finally, to denote benchmark values we use an overline, so that $\bar{t}_{ir}^{pd}$ represents the benchmark value of t_{ir}^{pd}.

3.1.1 Sectoral production (`Y(j,r)`)

Sectoral production combines intermediate inputs with a value-added nest combining primary inputs (see Figure 2). The unit cost of value-added is a CES composite of skilled and unskilled labor, land, resources and capital inputs to production, gross of taxes. Factor inputs

[13] To retain consistency with the MCP format, we express zero profit conditions as "oriented equations," with marginal cost on the LHS and marginal revenue on the RHS.

may be sector-specific or mobile across sectors:

$$p_{fjr}^{pf} = \begin{cases} p_{fr}^{F} \frac{(1+t_{fjr}^{f})}{1+\bar{t}_{fjr}^{f}} & f \in m_f \\ p_{fjr}^{S} \frac{(1+t_{fjr}^{f})}{1+\bar{t}_{fjr}^{f}} & f \in s_f \end{cases}$$

and the unit cost function is given by:

$$c_{jr}^{f} = \left(\sum_{f} \theta_f \left(p_{fjr}^{pf} \right)^{1-\sigma} \right)^{1/(1-\sigma)}$$

The user cost of intermediate inputs differs from the market price due to the presence of taxes on intermediate inputs:

$$p_{ijr}^{d} = p_{ir}^{Y} \frac{1+t_{ijr}^{fd}}{1+\bar{t}_{ijr}^{fd}}$$

$$p_{ijr}^{i} = p_{ir}^{M} \frac{1+t_{ijr}^{fi}}{1+\bar{t}_{ijr}^{fi}}$$

A CES cost function describes the minimum cost of a bundle of domestic and imported inputs to production, based on benchmark value shares and an elasticity of substation $\sigma = esubd_i$:

$$c_{ijr}^{i} = \left(\theta_d (p_{ijr}^{d})^{1-\sigma} + (1-\theta_d)(p_{ijr}^{i})^{1-\sigma} \right)^{1/(1-\sigma)}$$

Unit cost of sectoral output is then a Leontief (linear) composite of the costs of intermediate and value-added composite inputs, based on base-year value shares:

$$c_{jr}^{Y} = \sum_{i} \theta_i c_{ijr}^{i} + \theta_f c_{jr}^{f}$$

Having formulated the unit cost function, it is possible to compactly portray the zero profit condition for y_{jr}. In equilibrium, the marginal cost of supply equals the market price, net of taxes:

$$c_{jr}^{Y} = p_{jr}^{Y} \frac{1-t_{jr}^{o}}{1-\bar{t}_{jr}^{o}} \tag{8}$$

In the SOE model production for domestic and export markets are differentiated, and we replace p^Y_{jr} by a unit revenue function,

$$r^Y_{jr} = \left(\theta^D_{jr}(p^D_{jr})^{1+\eta} + (1-\theta^D_{jr})(p^X_{jr})^{1+\eta}\right)^{1/(1+\eta)}.$$

3.1.2 Consumer demand (`Y("c",r)`)

In previous versions of GTAPINGAMS the consumer price index represented a Cobb-Douglas demand system with an Armington substitution between domestic and imported goods prices gross of tax. This model is retained as one option in the new model, price indices for domestic and imported goods are given by:

$$p^{dc}_{ir} = p^Y_{ir}\frac{1+t^{pd}_{ir}}{1+\bar{t}^{pd}_{ir}},$$

and

$$p^{ic}_{ir} = p^M_{ir}\frac{1+t^{pi}_{ir}}{1+\bar{t}^{pi}_{ir}},$$

and the Armington composite price of good i is a CES composite price defined over these indices

$$p^c_{ir} = \left(\theta(p^{dc}_{ir})^{1-\sigma} + (1-\theta)(p^{ic}_{ir})^{1-\sigma}\right)^{1/(1-\sigma)}.$$

In addition to the Cobb-Douglas representation of the final demand system, our core model includes a LES, which combines a Cobb-Douglas discretionary demand with a Leontief subsistence demand activity. The added complexity of the LES specification permits the model to be calibrated both to the income and *average* price elasticities of demand. The price indices for discretionary and subsistence demand are defined by:

$$p^{DD}_r = \prod_i (p^c_{ir})^{\theta^C_{ir}}, \tag{9}$$

while the price index for subsistence demand is defined by a Leontief cost function:

$$p^{SD}_r = \sum_i \theta^{SD}_i p^c_{ir}. \tag{10}$$

Finally, GAMS code includes a third representation of final demand in the form of the CDE demand system (Hanoch, 1975). This representation has been part of the GTAP model in

GEMPACK for many years, and our implementation follows those of Hertel and van der Mensbrugghe (2015) and Chen (2015). Similar to the LES demand system, this approach permits calibrating empirical evidence on both own-price and income elasticities. Some details of CDE implementation and calibration are described in Appendix C.

3.1.3 Government demand (`Y("g",r)`)

Public expenditure is a fixed-coefficient aggregate of Armington composite goods. Within each composite domestic and imported goods trade off with a constant elasticity of substitution. The unit price indices for domestic and imported goods are given by:

$$p_{ir}^{dg} = p_{ir}^{Y} \frac{1 + t_{ir}^{dg}}{1 + \bar{t}_{ir}^{gd}}$$

and

$$p_{ir}^{ig} = p_{ir}^{M} \frac{1 + t_{ir}^{ig}}{1 + \bar{t}_{ir}^{ig}}$$

The composite price of the ith good is then:

$$pg_{ir} = \left(\theta (p_{ir}^{dg})^{1-\sigma} + (1-\theta)(p_{ir}^{ig})^{1-\sigma} \right)^{1/(1-\sigma)}$$

The unit cost of public services (G_r) is defined by the Leontief cost coefficients:

$$\sum_i \theta_i pg_{ir} = p_r^G \,. \tag{11}$$

3.1.4 Aggregate imports (`M(i,r)`)

Import cost index applies export taxes, trade and transport margins and import tariffs to the producer supply prices in exporting regions:

$$py_{isr}^{m} = p_{is}^{Y} \frac{(1 - t_{isr}^{xs})(1 + t_{isr}^{ms})}{(1 - \bar{t}_{isr}^{xs})(1 + \bar{t}_{isr}^{ms})} \,.$$

In the case of the SOE model, the supply price p^Y_{is} is replaced by export price p^X_{is}. The unit price of transportation services is given by:

$$pt^m_{jisr} = p^T_j \frac{1 + t^{ms}_{isr}}{1 + \bar{t}^{ms}_{isr}}.$$

Transportation margins enter as fixed coefficients with bilateral trade flows, so the unit delivered price is a convex combination of the unit prices with weights corresponding to base year value shares:

$$pyt^m_{isr} = \theta py^m_{isr} + \sum_j \theta^T_j pt^m_{jisr}.$$

Having formed a price index for bilateral imports from region s to region r, the CES cost index can be defined on the basis of value shares and the elasticity of substitution across imports from different regions, $\sigma = esubm_i$:

$$cim_{ir} = \left(\sum_s \theta_s (pyt^m_{isr})^{1-\sigma)} \right)^{1/(1-\sigma)}.$$

The import activity (m_{ir}) has a zero profit condition which relates the unit cost of imports to the market price of the import aggregate:

$$cim_{ir} = p^M_{ir}. \tag{12}$$

3.1.5 International transportation services (`YT(j)`)

For simplicity, the unit cost of a transportation service depends on the benchmark value shares of region-specific services through a Cobb-Douglas cost function. Under perfect competition with free entry, the unit cost of international transport services equals the equilibrium market price:

$$\prod_r (p^Y_{jr})^{\theta_j} = p^T_j \tag{13}$$

In the SOE model, the supply price p^Y_{is} in equation is replaced by export price p^X_{is}.

3.1.6 Sector-specific factor transformation (`FT(f,r)`)

The unit value of sector-specific factors is defined as a CET revenue function based on the base year value shares (θ_j):

$$pvfm_{fr} = \left(\sum_j \theta_j ps_{fjr}^{1+\eta} \right)^{1/(1+\eta)} \qquad f \in s_f \, .$$

This defines the profit-maximizing allocation of factors to individual sectors. In equilibrium, the unit value of the aggregate factor is equal to the maximum unit earnings:

$$p_{fr}^S = pvfm_{fjr} \quad f \in s_f \tag{14}$$

3.2 Market clearance

Supply-demand conditions apply to all goods and factors. Benchmark demand and supply quantities appear as scale factors in many of these equations, typically multiplied by activity levels which are equal to unity in the reference equilibrium.[14]

3.2.1 Firm output (`P(i,r)`)

Aggregate output of good i in region r in the reference equilibrium is `vom(i,r)`:

$$Y_{ir} vom_{ir} = \sum_j ddfm_{ijr} + ddfm_{iCr} + ddfm_{iIr} + ddfm_{iGr} + \sum_s dxmd_{irs} + dst_{ir} \tag{15}$$

[14] While not crucial for representation of the model as a nonlinear system of equations, we follow the MCP convention in writing out the market clearance conditions. The equations are "oriented", with supply variables on the LHS and demands on the RHS. Hence, the sense of the equation is $supply \geq demand$. In the core model equilibrium prices should always be positive, but in extensions of the standard model it might be quite common to introduce inequalities and complementary slackness, in which case the proper orientation of the equations is essential. Hence, in equilibrium should the price of a good be zero, economic equilibrium is then consistent with a market in which $supply > demand$.

where the compensated demand functions can be obtained by differentiating the unit cost functions:

$$
\begin{aligned}
ddfm_{ijr} &= Y_{jr} vdfm_{ijr} \left(\frac{ci_{ijr}}{p^d_{ijr}} \right)^{\sigma} \\
ddfm_{iCr} &= C_r \left(vdfm_{iCr} \frac{p^C_r}{pc_{ir}} \right) \\
ddfm_{iIr} &= I_r vdfm_{iIr} \\
ddfm_{iGr} &= G_r vdfm_{iGr} \\
dxmd_{isr} &= m_{ir} vxmd_{isr} \left(\frac{p^M_{ir}}{pyt^m_{isr}} \right)^{\sigma} \\
dst_{jr} &= YT_j vst_{jr} \frac{p^T_j}{p^Y_{jr}}
\end{aligned}
$$

In the SOE model ($etrndx_i < \infty$) equation (15) is replaced by two equations, one representing the market for domestic output and another representing the market for exports:

$$
\begin{aligned}
Y_{ir} ddm_{ir} &= \sum_j ddfm_{ijr} + ddfm_{iCr} + ddfm_{iIr} + ddfm_{iGr} && (16) \\
Y_{ir} dxm_{ir} &= \sum_s dxmd_{irs} + dst_{ir} \, . && (17)
\end{aligned}
$$

In these expressions, the compensated domestic and export supply coefficients are given by:

$$
\begin{aligned}
ddm_{ir} &= vdm_{ir} \left(\frac{r^Y_{ir}}{p^D_{ir}} \right)^{\eta} \\
dxm_{ir} &= vxm_{ir} \left(\frac{r^Y_{ir}}{p^X_{ir}} \right)^{\eta} .
\end{aligned}
$$

3.2.2 Private consumption (`P("c",r)`)

Consumer demand in region r in the reference equilibrium is vom_{Cr} hence:

$$
C_r vom_{Cr} = \frac{RA_r}{p^C_r} \qquad (18)
$$

3.2.3 Composite imports (`PM(i,r)`)

The aggregate value of imports of good i in region r in the reference equilibrium is vim_{ir}:

$$M_{ir} vim_{ir} = \sum_j difm_{ijr} + difm_{iCr} + difm_{iGr} \tag{19}$$

where compensated demand functions are given by:

$$\begin{aligned}
difm_{ijr} &= Y_{jr} vifm_{ijr} \left(\frac{ci_{ijr}}{p^i_{ijr}}\right)^{\sigma} \\
difm_{iCr} &= C_r vifm_{iCr} \left(\frac{pc_{ir}}{p^{ic}_{ir}}\right)^{\sigma} \frac{p^C_r}{pc_{ir}} \\
difm_{iGr} &= G_r vifm_{iGr} \left(\frac{pg_{ir}}{p^{ig}_{ir}}\right)^{\sigma} \frac{p^G_r}{pg_{ir}}
\end{aligned}$$

3.2.4 Transport services (`PT(j)`)

The aggregate demand (and supply) for transport service j in the benchmark equilibrium is vtw_j:

$$YT_j vtw_j = \sum_{isr} dtwr_{jisr} \tag{20}$$

where

$$dtwr_{jisr} = M_{ir} vtwr_{jisr} \left(\frac{p^M_{ir}}{pyt^m_{isr}}\right)^{\sigma}.$$

3.2.5 Primary factors (`PF(f,r)`)

The aggregate demand (and supply) of primary factor f in region r is $evom_{fr}$:

$$evom_{fr} = \begin{cases} \sum_j dfm_{fjr} & f \in m_f \\ evom_{fr} FT_{fr} & f \in s_f \end{cases} \tag{21}$$

where the demand for primary factor is given by:

$$dfm_{fjr} = Y_{jr} vfm_{fjr} \left(\frac{cf_{jr}}{p^{pf}_{fjr}}\right)^{\sigma}.$$

3.2.6 Specific factors (PS(f,j,r))

The net value of benchmark payments to factor f in sector j in region r is vfm(f,j,r):

$$vfm_{fjr}\left(\frac{p^S_{s_f,jr}}{p^F_{m_f,r}}\right)^{\eta} = dfm_{fjr} \tag{22}$$

where the demand for primary factor is written above.

3.3 Regional budget (RA(r))

Private and public incomes are given by :

$$RA_r = \sum_f p^F_{fr} evom_{fr} + p^C_n vb_r - p^I_r vom_{Ir} - p^G_r vom_{Gr} + \mathcal{R}_r \tag{23}$$

The base year current account deficit in region r is vb(r), and region $r = n$ corresponds to the "numeraire region" who's consumption prices denominates international capital flows (following conventional static trade theory, we hold the current account deficit fixed in counterfactual analysis). Furthermore, tax revenue in region r consists of output taxes, intermediate demand taxes, factor taxes, final demand taxes, import tariffs and export subsidies:

$$\begin{aligned}\mathcal{R}_r = \; & \textstyle\sum_j \mathcal{R}^o_{jr} + \sum_{ij}\left(\mathcal{R}^{fd}_{ijr} + \mathcal{R}^{fi}_{ijr}\right) + \sum_{fj}\mathcal{R}^f_{fjr} \\ & + \textstyle\sum_i\left(\mathcal{R}^{pd}_{ir} + \mathcal{R}^{pi}_{ir} + \mathcal{R}^{gd}_{ir} + \mathcal{R}^{gi}_{ir}\right) - \sum_{is}\mathcal{R}^{xs}_{irs} + \sum_{is}\mathcal{R}^{ms}_{isr}\end{aligned} \tag{24}$$

Each of these components of tax revenue can be calculated as an ad-valorem or proportional tax rate times a market price times the quantity demanded or produced.

Taxes related to Y_{ir} include output taxes:[15]

$$\mathcal{R}^o_{jr} = t^o_{jr} vom_{jr} p^Y_{jr} Y_{jr} \quad [\texttt{REV\_TO(g,r)}],$$

tax revenue from intermediate inputs:

$$\mathcal{R}^{fd}_{ijr} = t^{fd}_{ijr} p^Y_{ir} ddfm_{ijr} \quad [\texttt{REV\_TFD(i,j,r)}],$$

[15] Tax revenues in the GAMS codes – both MCP and MGE are represented by the *macros* indicated in square brackets.

$$\mathcal{R}^{fi}_{ijr} = t^{fi}_{ijr} p^M_{ir} \mathit{difm}_{ijr}, \quad [\texttt{REV\_TFI(i,j,r)}],$$

and factor tax revenue:

$$\mathcal{R}^{f}_{fjr} = t^{f}_{fjr} p^F_{fr} \mathit{dfm}_{fjr} \quad [\texttt{REV\_TF(f,g,r)}].$$

Taxes on household consumption of domestic and imported goods are:

$$\mathcal{R}^{pd}_{ir} = t^{pd}_{ir} p^Y_{ir} \mathit{ddfm}_{iCr}, \quad [\texttt{REV\_TFD(i,"C",r)}],$$

and

$$\mathcal{R}^{pi}_{ir} = t^{pi}_{ir} p^M_{ir} \mathit{difm}_{iGr} \quad [\texttt{REV\_TFI(i,"C",r)}].$$

Taxes on public demand for domestic and imported goods are:

$$\mathcal{R}^{gd}_{ir} = t^{gd}_{ir} p^Y_{ir} \mathit{ddfm}_{iGr} \quad [\texttt{REV\_TFD(i,"G",R)}],$$

and

$$\mathcal{R}^{gi}_{ir} = t^{gi}_{ir} p^M_{ir} \mathit{difm}_{iGr} \quad [\texttt{REV\_TFI(i,"G",r)}].$$

Export subsidies (paid by the government in the exporting region) are:

$$\mathcal{R}^{xs}_{irs} = t^{xs}_{irs} p^Y_{ir} \mathit{dxmd}_{irs} \quad [\texttt{REV\_TXS(i,r,s)}],$$

and import tariff revenues are given by:

$$\mathcal{R}^{ms}_{isr} = t^{ms}_{isr} \left(p^Y_{is}(1 - t^{xs}_{isr}) \mathit{dxmd}_{isr} + \sum_j p^T_j \mathit{dtwr}_{jisr} \right) \quad [\texttt{REV\_TMS(i,s,r)}].$$

4 Implementation and computer code

This section describes the implementation of GMR and SOE models in GAMS using the GTAP 9 dataset. We first describe the content of the distribution directory, and provide the logic of the model buildstream. We then discuss a number of issues related to data manipulation, namely aggregation across commodities and regions, data filtering and re-balancing. Finally, we discuss calibration of elasticities for international trade in GMR and SOE models, providing an intuitive (Marshallian) argument on how both models can be parametrized in order to approximate the same responsiveness to policy shocks.

4.1 Distribution Folders

We now overview the structure of the distribution directory. Practicalities on how to run the code is provided in Appendix B. Additional details about the directories and GAMS programs provided in GTAPINGAMS version 9 are provided in Appendix Appendix D. More detailed comments may also be found in comments interspersed in the GAMS code portions of which are presented in Appendix Appendix E.

The distribution directory contains eight second-level subdirectories, whose content we now overview in alphabetical sequence:

BUILD contains a command script which automates dataset construction. This script calls programs stored in the CODE subfolder, including FLEX2GDX.GMS, which translates data from .HAR to .GDX format,[16] FILTER.GMS, a program which filters and then rebalances the core dataset (see data filtering section below), GTAP9DATA.GMS, a program which reads a .GDX GTAPINGAMS dataset, GTAPAGGR.GMS, a program which performs dataset aggregation, and CDECALIB.GMS, a program which computes coefficients of the CDE demand system which are consistent with price and income elasticities of demand.

The distributed copy of `build.gms` reads, filters and aggregates data files for 2011. At the end of the computational process, listing files can be found in the BUILD directory, while the associated .GDX data files generated by these routines are saved in the GAMSDATA directory. See Appendix D for more details about the build process.

CODE contains the main GAMS program files which are included in this distribution. These files need not reside in this directory, but they are self contained and should always be moved as a collection. Two of the files in this directory contain GAMS code for the canonical static model, both specified as a mixed complementarity problem in two alternative representations:

[16] The .HAR format is used in the original distribution of the GTAP dataset. These data files need to be obtained separately and stored in the GTAPDATA subdirectory, which is discussed below.

MGE.GMS The standard model with tabular GAMS/MPSGE representation, and

MCP.GMS The standard model with an algebraic GAMS/MCP representation.

DEFINES contains mapping files for aggregations of GTAP 9 datasets. Files ending with .MAP define an aggregation in terms of the source dataset and mappings from sets in the source to sets in the target. Three single dimensional sets and three two-dimensional tuples are included in each mapping file. The sets and tuples are defined as follows:

```
set rr          Regions in the aggregation,
    ii          Commodities in the aggregation,
    ff          Primary factors in the aggregation,
    mapr(r,rr)  Mapping of source to target regions (from-to),
    mapi(i,ii)  Mapping from source to target commodities
    mapf(f,ff)  Mapping from source to target factors;
```

As an example, here is a set and mapping which aggregates skilled labor types:

```
set ff  Factors in the aggregated data /
        skl     Skilled workers,
        lab     Agricultural and unskilled workers,
        lnd     Land,
        cap     Capital,
        res     Natural resources /;

set mapf(f,ff)  Factor aggregation assignments /
        mgr.skl Officials and Mangers legislators (ISCO-88 Major Groups 1-2),
        tec.skl Technicians technicians and associate professionals
        clk.skl Clerks
        srv.lab Service and market sales workers
        lab.lab Agricultural and unskilled workers (Major Groups 6-9)
        lnd.lnd Land,
        cap.cap Capital,
        res.res Natural resources /;
```

Four alternative regional / commodity aggregation are included in the DEFINES directory to illustrate how aggregation works in GTAPINGAMS. The first is the full disaggregate database GTAPINGAMS.MAP which retains all regions and markets from the GTAP database after original data have been filtered and a few sector labels have been changed.[17]

Three aggregated datasets are based on regional disaggregation which distinguishes the G20, other oil exporters, a composite region representing other low income coun-

[17] The sectoral identifier which differ from the GEMPACK model are as follows: OIL (crude oil) is relabeled CRU, COA (coal) becomes COL, P_C (petroleum and coal products) becomes OIL, ELY (electricity) becomes ELE, and ELE (electronic equipment) becomes EEQ.

tries and a composite region representing other middle income countries. The G20_IEA aggregates commodities into 23 sectors included in the International Energy Agency energy database, and G20_MACRO aggregates to a four commodity sectoral mapping distinguishing agriculture, manufacturing, services and energy.

DOC contains the paper you are reading (please check the version).

FORENSICS contains GAMS programs illustrating how the distribution can be used to carry out policy analysis. Specifically, we provide code to evaluate the sensitivity of unilateral trade tax reform with respect to final demand system and model closure. Program RUNS.GMS generates RUNS.BAT, a Windows batch command file for systematic sensitivity analysis. This executes a sequence of calculations under alternative closures and policy region, each corresponding to an invocation of the scenario file TGRID.GMS. Results from the sequence of simulations are merged into a PivotChart dataset in an Excel workbook.

GAMSDATA contains the datasets generated by running `build.gms` in the BUILD directory. More specifically, files that are created and stored in this directory have a GAMS data-exchange (*.GDX) extension and are constructed data files that are either used as the source data for aggregation (e.g. GSD.GDX) or the target data emerging from the data aggregation (e.g. the fill GTAP data stored as GTAP9INGAMS.GDX), data files which are then then used in conjunction with the model files). Here are the sizes of the 2011 datasets produced for a 2011 base year:

```
gsd.gdx              43,910,349     ! Produced by flex2gdx.gms
gsd_5.gdx            26,690,775     ! Produced by filter.gms -- nd=5
gtap9ingams.gdx      25,923,318     ! Aggregated from gsd_5 to relabel
g20.gdx               3,770,394     ! Aggregated from gtap9ingams.
g20_iea.gdx             992,710     ! Aggregated from g20 (23 sectors)
g20_macro.gdx           113,224     ! Aggregated from g20 (4 sectors)
```

GTAPDATA contains `.zip` files from the GTAP distribution (Flexagg package). These files are not provided in the GTAPINGAMS distribution, but must be copied here by the user. They may include any or all of these files:

```
23-Aug-2016   2:57:22p     77,616,861   flexagg9aY04.zip
23-Aug-2016   2:57:38p     80,406,868   flexagg9aY07.zip
23-Aug-2016   2:57:08p     80,056,730   flexagg9aY11.zip
```

TEST contains programs which provide a consistency test of models. See Appendix D for a description of these programs.

4.1.1 Aggregation across commodities

Any GTAPINGAMS dataset may be aggregated into fewer regions, sectors and primary factors. This permits a modeler to do preliminary model development using a small dataset to ensure rapid response and a short debug cycle. After having implemented a small model, it is then a simple matter to expand the number of sectors and/or regions in order to obtain a more precise empirical estimate.

Conveniently, all GTAP datasets are defined in terms of three primary sets: i, the set of sectors and produced commodities, r the set of countries and regions, and f the set of primary factors. Table A1 presents the identifiers for the 59 GTAP 9 sectors in their most disaggregate form (see Appendix A). These sectors may be aggregated freely to produce more compact dataset.

Aggregation across commodities involves the reclassification of goods at *a benchmark point in the price space*. If we index the *disaggregate* goods using i, j and index the corresponding *aggregated* goods with I, J, we can identify commodities combined to form I as $i \in \mathcal{I}_I$. Aggregation at reference prices $\bar{p}_i$ implies that the value of aggregate demand equals the value of the constituent disaggregate demand:[18]

$$\bar{p}_I C_I(\bar{p}) = \sum_{i \in \mathcal{I}_I} \bar{p}_i C_i(\bar{p}). \tag{25}$$

Furthermore, *consistent aggregation* implies that both demand and the elasticity of demand at the benchmark point in the aggregate model coincides with the demand response in the disaggregate model, taking benchmark prices into account. Formally:

$$\bar{p}_J \left. \frac{\partial C_I}{\partial p_J} \right|_{\bar{p}} = \sum_{i \in \mathcal{I}_I, j \in \mathcal{I}_J} \bar{p}_j \left. \frac{\partial C_i}{\partial p_j} \right|_{\bar{p}}. \tag{26}$$

[18] We use $\bar{p}_i$ and $\bar{p}_I$ in this expression to emphasize the role of price as an aggregation weight, but without loss of generality, we may assume that the reference price vectors $\bar{p}_i = 1 \quad \forall i$ and $\bar{p}_I = 1 \quad \forall I$. This Harberger normalization greatly simplifies notation. When units are scaled so that the unit cost at this point is one, values shares at this point are identically equal to share parameters in the CES function, i.e. $\alpha_i = \theta_i$, and benchmark demands are equal to value shares, i.e. $\bar{C}_i = \theta_i$.

An implication is that the Allen-Uzawa elasticity of substitution (AUES) matrix of the aggregate model has to correspond to valueshare-weighted average of terms in the AUES matrix of the disaggregate model:

$$\hat{\sigma}_{IJ} = \sum_{i \in \mathcal{I}_I, j \in \mathcal{I}_J} \frac{\theta_i}{\hat{\theta}_I} \frac{\theta_j}{\hat{\theta}_J} \sigma_{ij} \tag{27}$$

where $\hat{\theta}_I, \hat{\theta}_J$ are the value share of inputs I and J in the aggregate cost function, and $\hat{\sigma}$ is the AUES matrix in terms of the aggregate prices.[19]

In practice, our aggregation routine proceeds as follows. First, we construct the full AUES matrix from the CDE parameters at the benchmark point ($\texttt{subpar}_{ir}$ in the GTAP database), using expression (33) in Appendix C. Second, to ensure a consistent demand response when the data is aggregated across commodities, we use the aggregated value shares and the diagonal terms of the AUES matrix to recalibrate coefficients of the CDE demand system (as described in Appendix C).

[19] Note that exact aggregation based on (27) is only possible when working with a specific functional form, in which case own-price elasticities in the aggregated model are:

$$\epsilon_I = \hat{\theta}_I \sigma_{II} = \hat{\theta}_I \sum_{(ij) \in \mathcal{I}_I} \frac{\theta_i}{\hat{\theta}_I} \frac{\theta_j}{\hat{\theta}_I} \sigma_{ij} \tag{28}$$

Note also that if we had information about benchmark value shares (θ_i) and benchmark elasticities of demand (ϵ_i), but no information regarding off-diagonal terms in the AUES matrix σ_{ij}, $i \neq j$, the vectors θ_i and ϵ_i may or may not be consistent with a given cost function. In this setting, aggregation can use a simplifying assumption that the average off-diagonal AUES in the aggregation-relevant terms equals the overall average off-diagonal value for each of the constituent goods. From this it follows that average share-weight off-diagonal AUES value associated with input i is given by:

$$\bar{\sigma}_i = \frac{\epsilon_i}{\theta_i - 1}$$

Substituting $\bar{\sigma}_i$ for σ_{ij} in (28), we have a formula for an inexact aggregation:

$$\hat{\epsilon}_I = \sum_{i \in \mathcal{I}_I} \left(\frac{1 - 1/\hat{\theta}_I}{1 - 1/\theta_i} \right) \frac{\theta_i}{\theta_I} \epsilon_i \tag{29}$$

In either exact aggregation (working from a full AUES matrix) or inexact aggregation (based on average off-diagonal values), we find that $\theta_i < \theta_I \quad \forall i \in \mathcal{I}_i$, hence the elasticity of demand for composite good I is less than a weighted sum of the elasticities of demand in the constituent goods:

$$|\hat{\epsilon}_I| < \sum_{i \in \mathcal{I}_I} \frac{\theta_i}{\hat{\theta}_I} |\epsilon_i| \,.$$

4.1.2 Aggregation across regions

Suppose that we aggregate across regions as well as across commodities.[20] Cost functions in regions r and r' are disjoint, so the demand response for p_{ir} does not depend on $p_{ir'}$. We therefore begin by calculating price elasticities for the aggregate goods within each region, and then produce a quantity-weighted aggregation of elasticities across regions. Therefore, the own-price elasticity of demand for aggregate commodity I in aggregate region R is given by:

$$\epsilon_{IR} = \frac{\sum_{r \in \mathcal{R}_R} \epsilon_{Ir} \bar{C}_{Ir}}{\sum_{r \in \mathcal{R}_R} \bar{C}_{Ir}},$$

where $\bar{C}_{Ir}$ is benchmark consumption. As noted in footnote 18, because of the Harbgerger normalization, it is also equal to the value shares.

Own-price elasticities together with value shares define diagonal terms in the Allen-Uzawa elasticity matrix. To compute off-diagonal AUES terms, we use a value-weighted aggregation of cross-price elasticities across regions. In particular, cross-price (compensated) elasticities are given by:

$$\epsilon_{IJr} = \frac{\sum_{i \in \mathcal{I}_I, j \in \mathcal{I}_J} \sigma_{ijr} \bar{C}_{ir} \bar{C}_{jr}}{\sum_{i \in \mathcal{I}_I, j \in \mathcal{I}_J} \bar{C}_{ir} \bar{C}_{jr}} \, \theta_{Jr}.$$

To ensure that these terms are consistent with the adding up condition of the AUES matrix, we average these terms across constituent regions and scale them approproately:

$$\sigma_{IJR} = \lambda_{IR} \left(\frac{\sum_{r \in \mathcal{R}_R} \epsilon_{IJr} \; \psi_{IJrR}}{\theta_{JR}} \right) \qquad \forall I \neq J \,,$$

in which

$$\psi_{IJrR} = \frac{\bar{C}_{Ir} \bar{C}_{Jr}}{\sum_{r' \in \mathcal{R}_R} \bar{C}_{Ir'} \bar{C}_{Jr'}},$$

and the scaling factor λ_{IR} is chosen so that $\sum_J \sigma_{IJR} \; \theta_{JR} = 0$. Given our somewhat ad-hoc treatment of off-diagonal terms, it seems sensible to define aggregations from the GTAP dataset with full commodity and regional details rather than from a dataset which has already been aggregated.

[20] Note that regional identifiers in the full dataset correspond to ISO alpha-3 country codes. Users can define their own aggregations of the GTAP data and use any labels to describe regions. For technical reasons, if a GTAP dataset is to be used with MPSGE, then *regional identifiers can have at most 4 characters*. Table A2 presents the three-character identifiers which are normally used for primary factors.

4.2 Data filtering

The GTAP source data in original form presents substantial challenges for calibrated models processed using direct solution methods (e.g., PATH, CONOPT or IPOPT). In our experience, most numerical problems with GTAPINGAMS models can be traced to density of the source data in which we find large numbers of small coefficients. These coefficients portray economic flows which are a negligible share of overall economic activity, yet impose a significant computational burden during matrix factorization.

In order to "filter" these economically insignificant value and reduce dimensionality of the problem, GTAPINGAMS includes a GAMS program (FILTER.GMS) which removes small values which are smaller than a specified tolerance level. An input to this program (ND) determines the filter tolerance, i.e. the number of decimals for the smallest coefficient to be retained in the data. For example, when $nd = 4$, the smallest coefficient in the benchmark social accounting matrix is 0.0001. When $nd = 6$, the smallest number is 0.000001, etc. Larger values of ND retain a larger number of small coefficients in the filtered dataset. Filtering makes a GTAP database smaller, as illustrated in Table 6 in which it can be seen that filtering reduces the size of a GTAP database by somewhere between 20% and 50%, depending on the filtering tolerance.

Most of the reduction in non-zeros results from the elimination of small intermediate inputs and bilateral trade flows. Importantly, the filtering procedure has differential impacts on regions and markets in the database, and the choice of a filter tolerance depends to some extent on the size of the countries to be included in a given analysis. For our analysis of the G20 countries in the present paper, a filter tolerance $nd = 5$ seems suitable, whereas $nd = 7$ leads to numerical problems due to small coefficients. Therefore, depending on the nature of the policy question under investigation, different filtering strategies may be adopted. Individual researchers may have their own opinions about how to select parameter values should be rounded to zero. The version of `filter.gms` provided with the GTAPINGAMS distribution is intended to provide a starting point for this step in the dataset development process.

Removing small entries from the dataset implies that the resulting filtered dataset no longer represent a micro-consistent matrix. However, unlike earlier versions of GTAPINGAMS, we do not use a nonlinear optimization framework to rebalance the data. Instead, `filter.gms`

Table 6: Filtering results for entire dataset (2011 data)

		Nonzeros	Nonzeros After Filtering			% Reduction in Nonzeros		
		Before	$nd = 4$	$nd = 5$	$nd = 6$	$nd = 4$	$nd = 5$	$nd = 6$
Domestic Intermediate	*vdfm*	478800	233267	290668	342177	51	39	29
Imported Intermediate	*vifm*	478798	186521	247571	300255	61	48	37
Bilateral trade	*vtwr*	776358	175449	314173	463832	77	60	40
Transport margins	*vxmd*	1097628	400882	591129	766803	63	46	30

moves imbalances resulting from omitted coefficients into either factor supplies or investment demand depending on the sign of imbalance which appears following filtering. This approach to reconciliation is simple to implement provided that the inconsistencies resulting from filtering are small. The reconciliation methodology implemented in `filter.gms` is perhaps less useful for large scale recalibration exercises such as might arise with a whole change in benchmark tax rates or production structure.

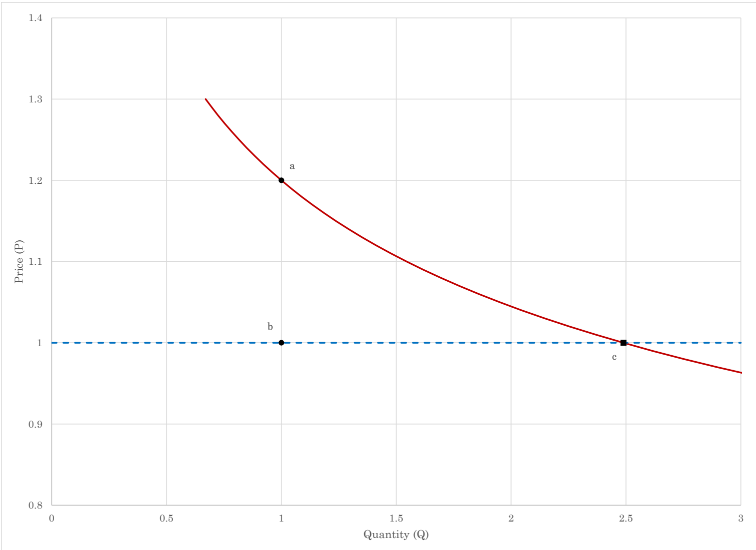
In practice, we note that for $nd = 5$, the data rebalancing procedure imposes very small changes in the remaining nonzero elements, but it substantially reduces the number of coefficients in the dataset as indicated in Table 6. Nearly half of the trade flow and imported intermediate demand coefficients can be dropped, resulting in a reduction in the size of the dataset from nearly 44 MB to 27 MB. More importantly, sparsity tends to improve robustness. Larger models are easier to solve when the underlying datasets have been carefully filtered.

4.3 Calibration of trade elasticities

The objective of this section is to outline a strategy to parametrize CET elasticity of transformation between domestic production and exports (`etadx(g)`) and elasticity of substitution between domestic production and imports (`esubdm(i)`). Intuitively, in the SOE model, a finite export supply elasticity is specified, the import demand elasticity is adjusted to maintain a consistent with the empirical elasticity estimates in the GTAP dataset. This ensures that aggregate trade response is consistent in both GMR and SOE models.

For simplicity, we approach the problem from a Marshallian perspective. Suppose that on a given trade flow, the import demand elasticity is $\bar{\epsilon}$, and benchmark rate of protection is $\bar{t}$. Scaling units so that the FOB supply price is 1, the demand price is $1 + \bar{t}$, and the quantity demanded in the benchmark is 1.

Figure 9: Market Clearance $\epsilon = 5$, $\eta = \infty$



In a setting in which the supply elasticity of exports, $\bar{\eta}$, is infinite, which is aligned with assumptions underlying our SOE representation, the supply price of exports is constant ($p = 1$). Therefore, if the tariff were removed, and demand is isoelastic, the new equilibrium quantity and price are given by:

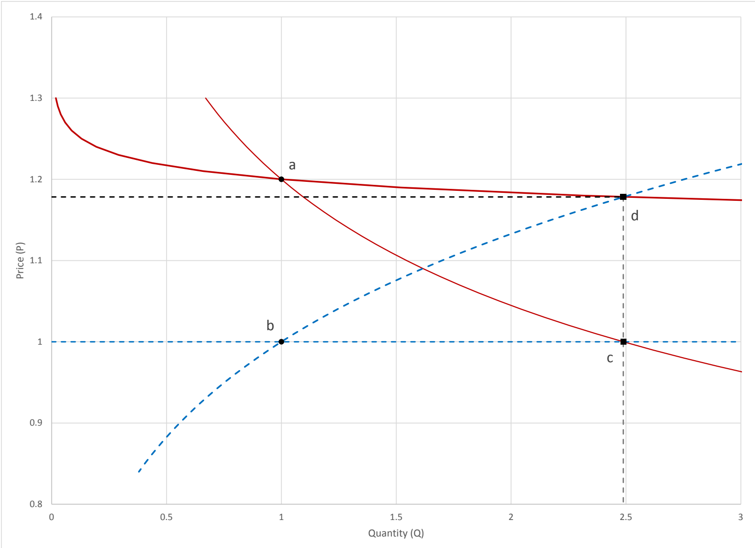
$$q^* = (1 + \bar{t})^{|\bar{\epsilon}|} \ . \tag{30}$$

Under this free trade outcome with constant supply price, the quantity demanded at $p = 1$ is the new equilibrium quantity. This outcome is illustrated in Figure 9, in which benchmark import demand is at price-quantity point a, benchmark import supply is at point b, and there is a benchmark trade barrier equal to 20%. The free-trade equilibrium is at point c, reflecting benchmark import demand elasticity $\bar{\epsilon} = 5$.

Now instead of a perfectly elastic supply function, suppose that we consider $\eta < \infty$ as is the case in the GMR model. In this case, removing tariffs implies a new equilibrium supply price p^*, and the associated equilibrium supply and demand q^* from (30) is given by:

$$q^* = \left(\frac{1 + \bar{t}}{p^*} \right)^{|\epsilon|} = (p^*)^{\eta} \ .$$

Figure 10: Market Clearance $\epsilon = 16.7$, $\eta = 7.1$



This expression shows that in order for the aggregate trade response under $\eta < \infty$ to be equal to the one prevailing under $\eta = \infty$, demand elasticity (ϵ) has to be adjusted as the export supply elasticity η is reduced. Formally, given equilibrium price:

$$p^* = (1 + \bar{t})^{|\bar{\epsilon}|/\eta},$$

the reparametrized elasticity of demand is given by:

$$|\epsilon| = \frac{|\bar{\epsilon}|}{1 - |\bar{\epsilon}|/\eta}.$$

Let $\theta = 1 - |\bar{\epsilon}|/\eta$. Consistency of trade quantity responses in the revised model implies that $\theta \geq 0$. When $\eta = \infty$, then we have the original model with a horizontal supply curve. Setting $\theta = 0.3$, the new Marshallian model appears in Figure 10.

As in the previous model, a and b are benchmark demand and supply with a 20% tariff barrier. While free trade in the original model is at point c, free trade in the revised model is at point d. Supply is less elastic, so social surplus increases are captured by the supply side of the market, and the equilibrium price following tariff removal increases by over 10% from the benchmark level.

In the GAMS code, this adjustment takes place at the top of `gtap9data.gms` file. The run-time environment variable `thetadx` defines how the adjustment of export supply and import demand is to be calibrated to match the GMR GTAP model. The associated assignments are reported here.

```
parameter       etadx(g)        Export elasticity of transformation D vs E,
                esubdm(i)       Elasticity of substitution D vs M,
                vxm(g,r)        Export market supply;

$if not set thetadx $set thetadx 0.5

etadx(i)$(1-%thetadx%) = esubd(i)/(1-%thetadx%);
etadx(i)$(not (1-%thetadx%)) = +inf;
esubdm(i) = esubd(i)/%thetadx%;
vxm(i,s) = (vst(i,s) + sum(r,vxmd(i,s,r)))$(1/etadx(i));
```

5 A few forensic calculations

We conclude by reporting on a few calculations which illustrate of the importance of model closure for assessing the economic effects of policy reform. The GMR and SOE models are well suited to help understand the role of terms of trade in policy experiments. In these simulations, we consider on the economic impact of *unilateral, proportional* adjustment of regional import tariffs and export taxes. Furthermore, as a matter of illustration, we will focus only on results for the United States (USA) and China (CHN).[21]

Figure 11 summarizes the commodity-based taxes on imports to USA and CHN in the 2011 base year database. Taxes applied to USA imports are highest on wearing apparrel (WAP), textiles (TEX), and leather (LEA). In the GTAP9 database, these goods are subject to substantial export taxes. Among the highest taxed goods, the highest tariff revenue is associated with imports of chemicals, rubber and plastic products (CRP). Imports to China of sugar (SGR), "wool, silk-worm cocoons" (WOL), and "motor vehicles and parts" (MVH) are subject to tariffs of 20% or more, but the largest tariff revenue (among the highest taxed imports) are on "machinery and equipment" (OME), CRP and crude oil (CRU).

Figure 12 summarizes the region structure of taxes on imports to CHN in 2011. The

[21] Region CHN in our G20 aggregation includes GTAP regions CHN and HKG.

Figure 11: Trade taxes and tariff revenue by commodity

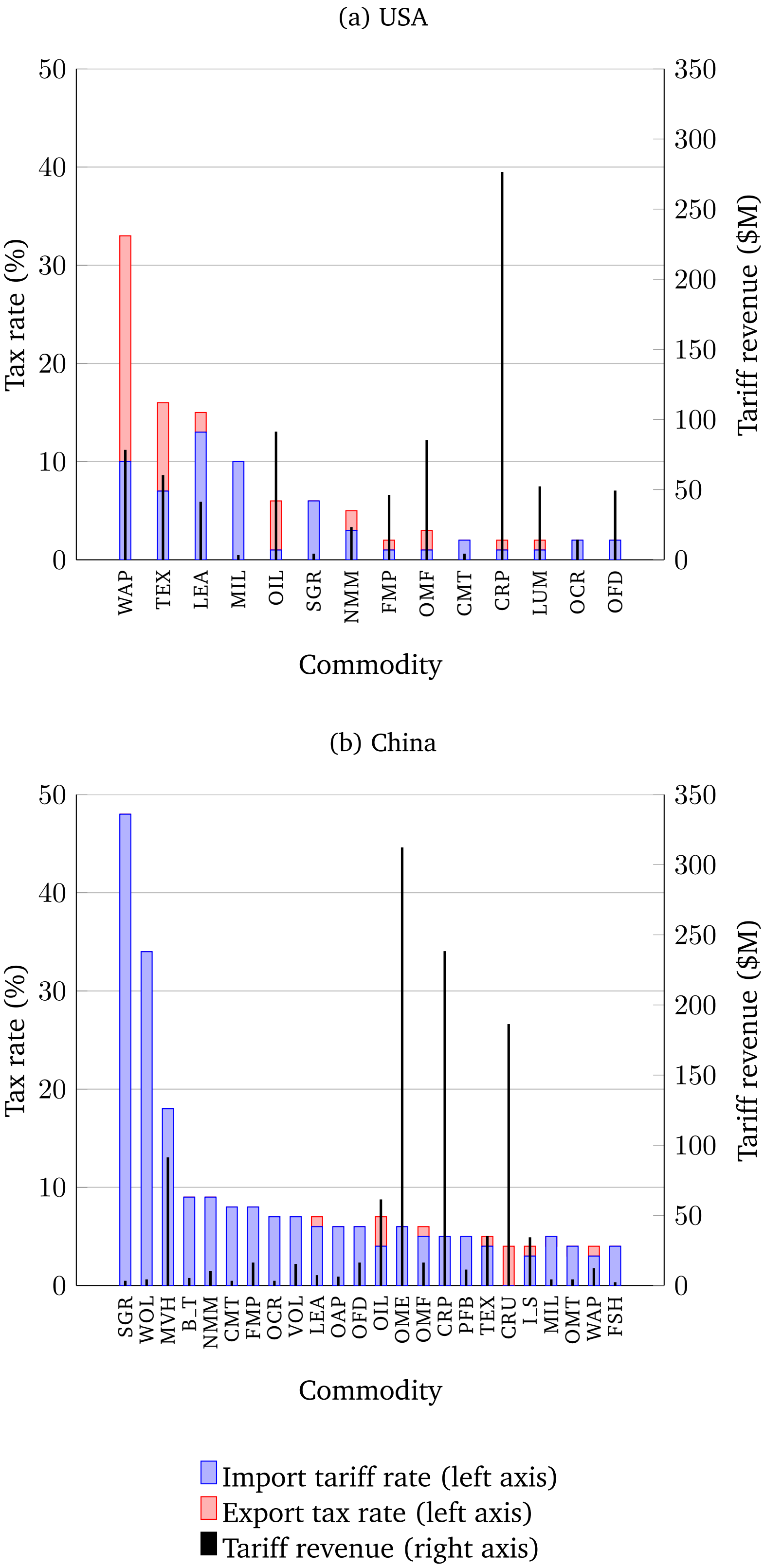
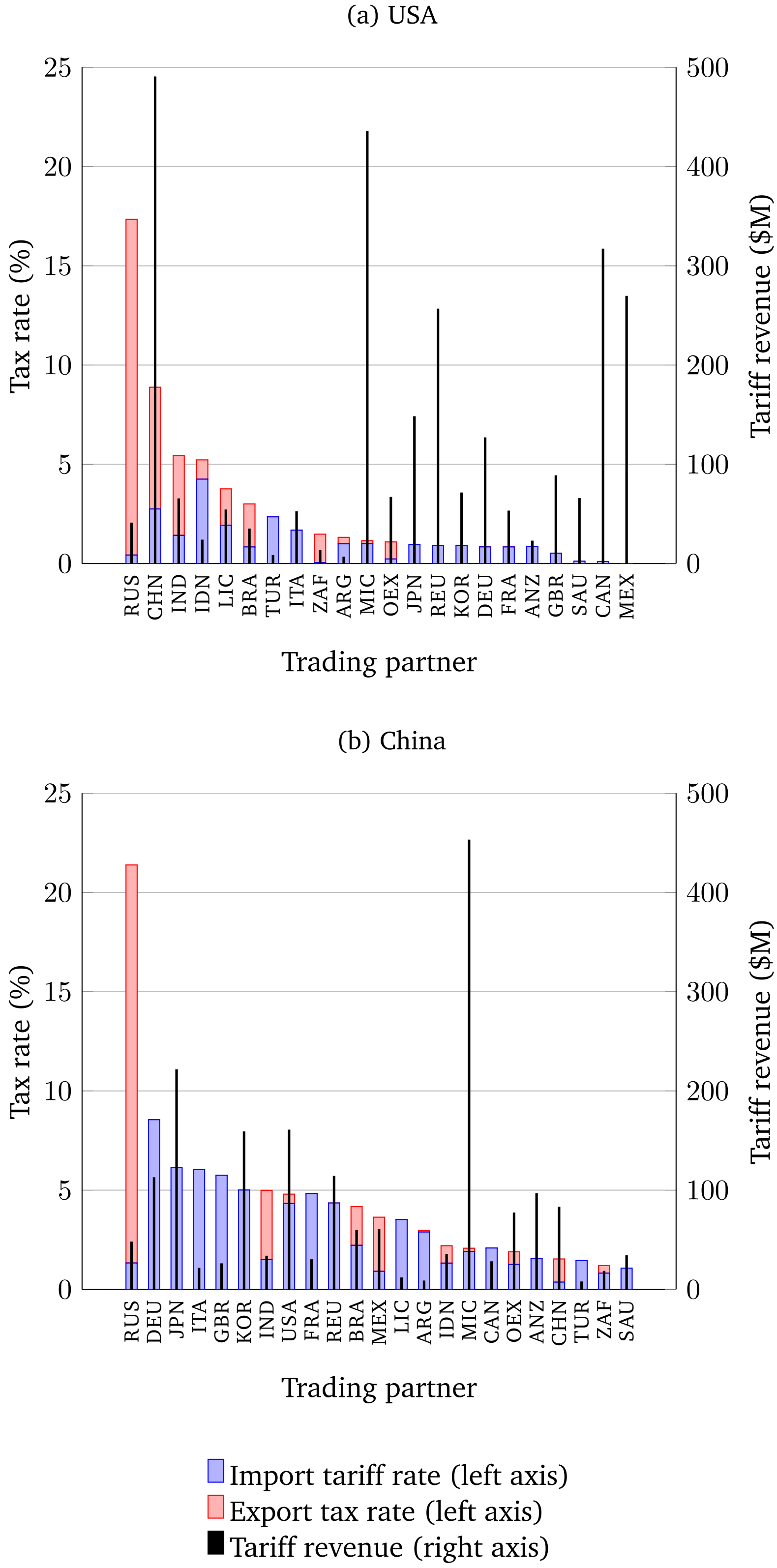


Figure 12: Trade taxes and tariff revenue by partner



highest trade taxes are applied on imports from Russia (RUS), China, India and Indonesia. Export taxes are the largest fraction of taxes on imports from Russia, China and India. Tariff revenue is concentrated on the countries which have the highest exports to the USA, China, Canada, Mexico and the European Union. Tariff rates are, however, quite low on these imports.

Taxes on imports to China are highest from Russia, again largely on the basis of export taxes applied by Russia. China's import tariffs average around 5% on CHN imports from the OECD coutries (Japan, European Union states, USA). The largest share of import tariff revenue is associated with imports from middle income countries (MIC). For the US, average taxes on imports (including both export taxes applied by trade parnters and import tariffs collected by USA are a little less than 2%). Taxes on imports to China average a bit more than 4%. We note that relative to other countries in the database, the US and China have relatively low trade taxes. Taxes in both the US and China are far from uniform, hence proportional increases in trade taxes might be expected to increase the distortionary cost of the existing tax system.

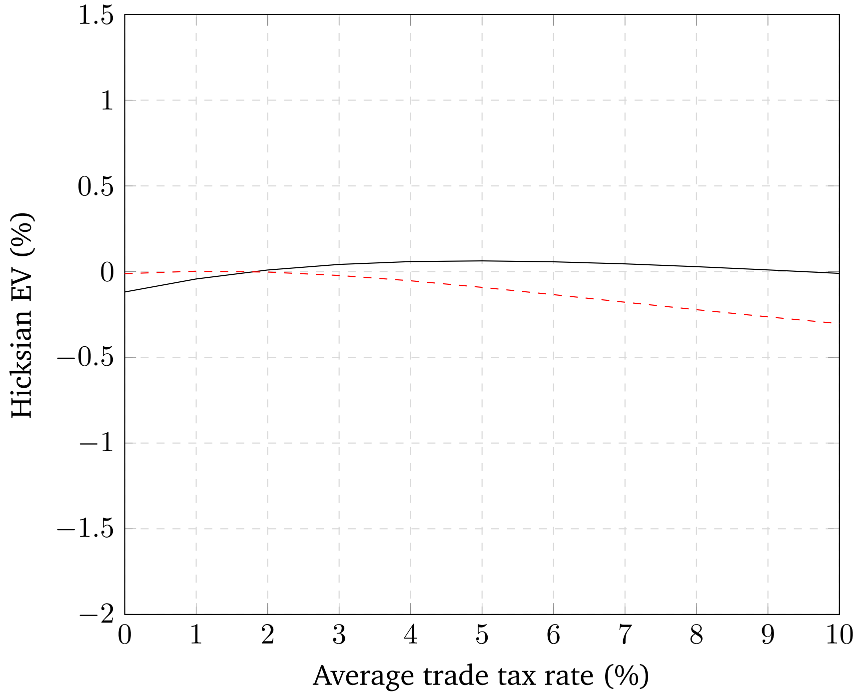
The results of our stylize policy experiment, in both the SOE and GMR models, are shown in Figure 13. We report the Hicksian equivalent variation as a function of the proportional adjustment of trade taxes, which ranges from zero (free trade) to a ten percent increase. Note that for these simulations, final demand is based on the CDE model. Results

In the GMR franework, the model suggest that the US can only slightly increase welfare by *increasing* trade taxes (up to a maximum of around 4.5%). By contrase, China increases welfare through a small *decrease* in trade taxes (to 3%). However, in our simulations the welfare impact depends whether one works with a multiregional or open economy closure. While in the GMR model China gains from a small reduction in tariffs, in the SOE model it gains the most from free trade. Therefore, although the GMR and SOE models are based on identical benchmark datasets, the absence of terms of trade effects in the open economy setting has significant implications for the effects of changes in tax policy.

In contrast to the importance of the model closure, the structure of final demand only modestly influences results for this specific set of simulations. This is illustrated in Figure 14. This is especially striking for the US, as the welfare effect of trade taxes is almost identical across alternative final demand specification. For China, differences are somewhat more

Figure 13: Welfare effect of unilateral policy

(a) USA



(b) China

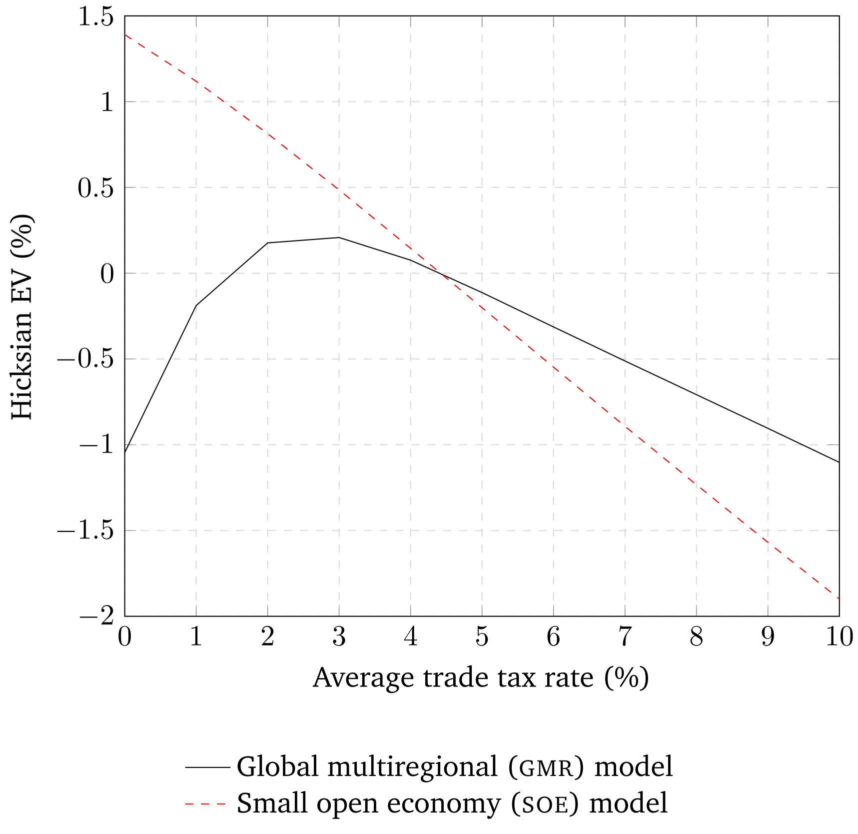
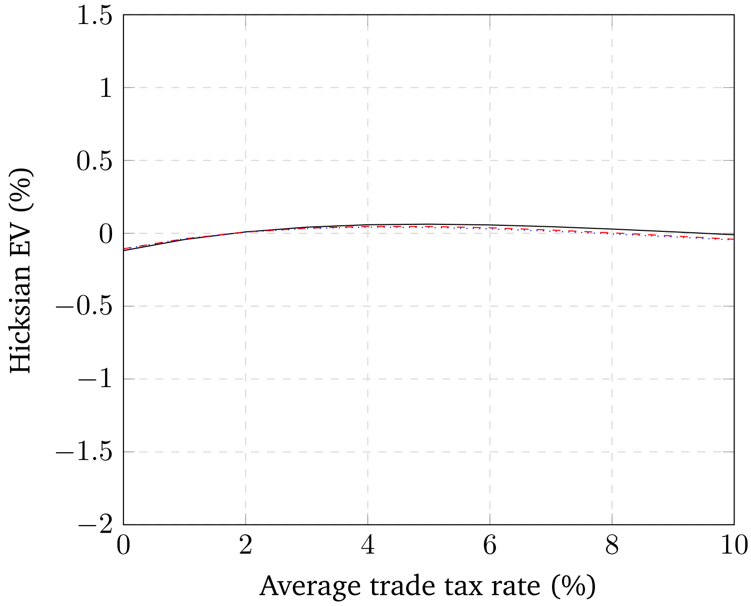
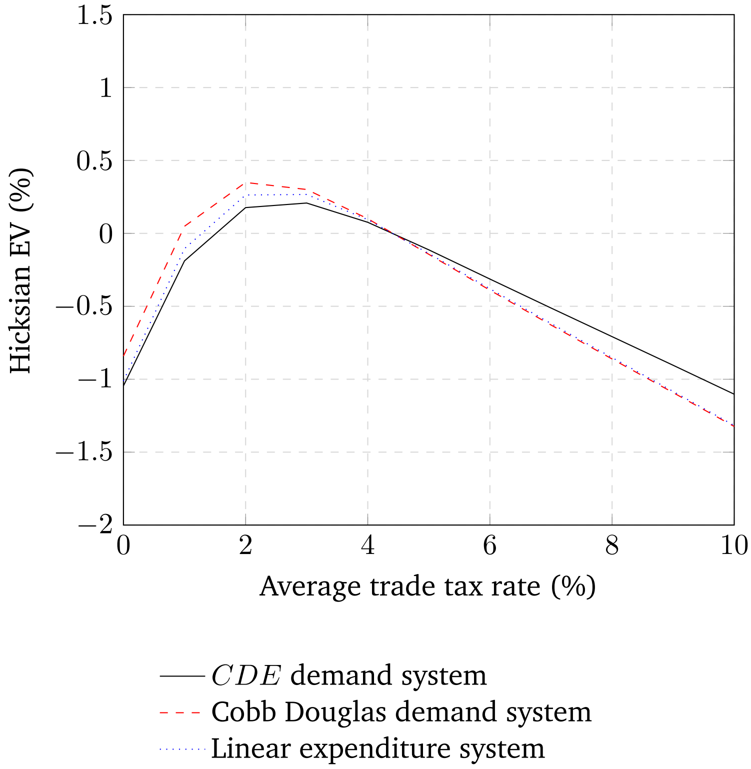


Figure 14: Demand system sensitivity

(a) USA



(b) China



pronounced, but they are quantitatively much less important than differences that can be attributed to alternative trade closures.

6 Concluding comments

This paper has documented two general models implemented in GAMS based on the GTAP version 9 database. The *global multiregional model* based on an Armington representation of bilateral trade flows in which terms of trade is endogenous, and a *small open economy model*, in which relative prices between distinguished regions are endogenous, but rest of world prices and the current account are fixed. These two models represent the "canonical" representations of international trades responses, one in which terms of trade are endogenous and another in which relative prices in rest-of-world are held fixed.

The model also integrates three different options for the representation of final demand. While the model based on Cobb-Douglas preferences is the most parsimonious, it cannot accommodate empirical evidence on own-price and income elasticity. This limitation is addressed by the two other alternatives, the *linear expenditure system* and the *constant difference of elasticities* model. For these models, however, there are additional challenges associated with the aggregation of the AUES matrix. We have discussed a possible approach to do so, but more work in this area seems necessary.

All in all, the set of models made available through this paper offer a very rich setup to study the quantitative impact of international policies. We stress that it is particularly important to study how alternative structural assumption affect the results. For the particular policy illustration we considered, our results suggest that trade closures matter a lot, whereas the representation of final demand is quantitatively less important. This will, of course, depend on the specific sort of policies considered.

Appendix A List of GTAP sectors and primary factors

Table A1: Commodities and Industries in the GTAP 9 database

Identifier	Definition	Identifier	Definition
PDR	Paddy rice	LUM	Wood products
WHT	Wheat	PPP	Paper products, publishing
GRO	Cereal grains nec	P_C	Petroleum, coal products
V_F	Vegetables, fruit, nuts	CRP	Chemical,rubber,plastic prods
OSD	Oil seeds	NMM	Mineral products nec
C_B	Sugar cane, sugar beet	I_S	Ferrous metals
PFB	Plant-based fibers	NFM	Metals nec
OCR	Crops nec	FMP	Metal products
CTL	Cattle,sheep,goats,horses	MVH	Motor vehicles and parts
OAP	Animal products nec	OTN	Transport equipment nec
RMK	Raw milk	ELE	Electronic equipment
WOL	Wool, silk-worm cocoons	OME	Machinery and equipment nec
FRS	Forestry	OMF	Manufactures nec
FSH	Fishing	ELY	Electricity
COA	Coal	GDT	Gas manufacture, distribution
OIL	Oil	WTR	Water
GAS	Gas	CNS	Construction
OMN	Minerals nec	TRD	Trade
CMT	Meat: cattle,sheep,goats,horse	OTP	Transport nec
OMT	Meat products nec	WTP	Sea transport
VOL	Vegetable oils and fats	ATP	Air transport
MIL	Dairy products	CMN	Communication
PCR	Processed rice	OFI	Financial services nec
SGR	Sugar	ISR	Insurance
OFD	Food products nec	OBS	Business services nec
B_T	Beverages and tobacco products	ROS	Recreation and other services
TEX	Textiles	OSG	PubAdmin/Defence/Health/Educat
WAP	Wearing apparel	DWE	Dwellings
LEA	Leather products	CGD	Aggregate investment

Table A2: Primary Factors in the GTAP 9 database

	Identifier	Definition
Mobile factors:		
	MGR	Officials and Managers legislators (ISCO-88 Major Groups 1-2),
	TEC	Technicians and associate professionals
	CLK	Clerks
	SRV	Service and market sales workers
	LAB	Agricultural and unskilled workers (Major Groups 6-9)
	CAP	Capital
Sector-specific factors:		
	LND	Land
	RES	Natural resources

Appendix B Practicalities

System Requirements

You will need to have the following:

- A computer.
- A GAMS system, Version 24.7.1 or newer.
- The PATH or CONOPT solvers.
- MPSGE subsystem (optional)

Getting Started

The GTAPINGAMS package is distributed as a zip file containing the directory structure and GAMS programs which can be unzipped into a clean working directory. GTAP source data are not distributed with the GTAPINGAMS system. In order to generate models with the GTAPINGAMS tools, it is necessary to obtain the GTAP 9 distribution archives.

Here are the steps involved in installing GTAPINGAMS:

1. Create an empty root directory for GTAP.
2. Unzip `gtap9ingams.zip` in this directory.
3. Install the GTAP data file `flexagg9aY11.zip` into the `gtapdata` subdirectory.[22]
4. Connect to the `build` directory and execute `gams build.gms` This will take some time to complete. The batch file is set up to begin with the 2011 base year data. Choose a different year if you wish.
5. Connect to the `test` directory and execute `run.gms` to assess benchmark consistency.
6. The GAMS programs in the `forensics` directory provide a template for how to conduct counterfactual policy simulations, using Excel PivotTables and PivotCharts to synthesize model output.

[22] You need only one data files, but these data are provided for three different years in the GTAP "flexhar" distribution.

Appendix C The CDE demand system

Theory and Analytics

Our implementation of the CDE model follows Chen (2015). In the CDE model, the utility index U is defined implicitly by the following equation:

$$\sum_i \theta_i U^{e_i(1-\alpha_i)} \left(\frac{P_i}{C}\right)^{1-\alpha_{ir}} = 1\,, \tag{31}$$

where C is the unit cost index, α_{ir} is the substitution parameter ($\texttt{subpar}_{ir}$ in the GTAP database), and e_{ir} is the income parameter ($\texttt{incpar}_{ir}$).

The unit cost (C) in the CDE model is determined by a budget constraint,

$$\sum_i P_i x_i = M$$

and demand functions can be derived using Roy's identify:[23]

$$x_i = \frac{\partial C/\partial P_i}{\partial C/\partial M} = \theta_i \frac{U^{(1-\alpha_i)e_i}(P_i/C)^{-\alpha_i}}{\sum_{j'} U^{(1-\alpha_j)e_j}(P_j/C)^{-\alpha_j}}$$

Here is a GAMS model which evaluates CDE demand across commodities i and regions $r \in \mathrm{rc}$. We have $n+2$ variables (x_i, C and U) determined by $n+2$ equations. While the CDE model is essentially a nonlinear system of equations, we write down the model using mixed complementarity syntax so as to associate equations with variables.[24]

```
variables       D(i,r)  Demand,
                C(r)    Price index,
                U(r)    Utility index;

equations       ddef, budget, choice;

ddef(i,rc(r))$theta(i,r)..
        D(i,r) * sum(j, theta(j,r) * U(r)**((1-alpha(j,r))*e(j,r))
                      *  (P(j,r)/C(r))**(1-alpha(j,r)))
                =e= theta(i,r)*U(r)**((1-alpha(i,r))*e(i,r))
                      * (P(i,r)/C(r))**(-alpha(i,r));

budget(rc(r))..
        sum(i, D(i,r)*P(i,r)) =e= M(r);
```

[23] The partial derivatives $\partial C/\partial P_i$ and $\partial C/\partial M$ are obtained by differentiating (31).

[24] If `U(r)` is fixed prior to a CDE solve, the corresponding `budget(r)` constraint is automatically dropped thereby providing the compensated demand response.

```
choice(rc(r))..
        sum(i, theta(i,r) * U(r)**((1-alpha(i,r))*e(i,r))
                        * (P(i,r)/C(r))**(1-alpha(i,r))) =e= 1;

model cdeeval /ddef.D, budget.U, choice.C/;
```

The price elasticity of demand in this model is given by:

$$\epsilon_i^{CDE} = \theta_i \left(2\alpha_i - \left(\sum_k \theta_k \alpha_k \right) \right) - \alpha_i$$

and the income elasticity of demand is given by:

$$\eta_i^{CDE} = \frac{e_i(1-\alpha_i) + \sum_k \theta_k e_k \alpha_k}{\sum_k \theta_k e_k} + \alpha_i - \sum_k \theta_k \alpha_k$$

Calibration

Calibration of the CDE parameters involves a least squares optimization model of the form:

$$\min_{\alpha,e} \sum_i \theta_i \left(\left(\epsilon_i^{CDE} - \epsilon_i\right)^2 + \left(\eta_i^{CDE} - \eta_i\right)^2 \right) \tag{32}$$

Here is a GAMS model which calibrates CDE demand across commodities i and regions $r \in \mathrm{rc}$:

```
variables
        E(i,r)          Substitution parameter,
        ALPHA(i,r)      Income parameter

        EPSILONV(i,r)   Calibrated own-price elasticity of demand,
        ETAV(i,r)       Calibrated income elasticity of demand;

equations       objcde, epsilondef, etadef;

objcde..        OBJ =e= sum((i,rb(r))$theta(i,r), theta(i,r) *
                        (sqr(EPSILONV(i,r) - epsilon(i,r))) +
                         sqr(ETAV(i,r)      - eta(i,r)));

epsilondef(i,rb(r))$theta(i,r)..
        EPSILONV(i,r) =e= theta(i,r) * (2*ALPHA_(i,r)-sum(k,theta(k,r)*ALPHA_(k,r)))
                                - ALPHA_(i,r);

etadef(i,rb(r))$theta(i,r)..
        ETAV(i,r) =e=
                (E(i,r)*(1-ALPHA_(i,r)) + sum(k,theta(k,r)*E(k,r)*ALPHA_(k,r))) /
                        sum(k,theta(k,r)*E(k,r))
                + (ALPHA_(i,r) - sum(k,theta(k,r)*ALPHA_(k,r)));

model cdecalib /objcde, epsilondef, etadef/;
```

Finally, we note that in the CDE model the Allen-Uzawa elasticities of substitution for compensated demand are given by:

$$\sigma_{ij}^{CDE} = \begin{cases} \alpha_i + \alpha_j - (\sum_k \theta_k \alpha_k) & i \neq j \\ 2\alpha_i - (\sum_k \theta_k \alpha_k) - \frac{\alpha_i}{\theta_i} & i = j \end{cases} \tag{33}$$

The "constant difference of elasticities" feature of the CDE demand system is evident through the calculation based on the AUES elasticities of sustitution:

$$\sigma_{ik} - \sigma_{jk} = \alpha_i - \alpha_j$$

Parallel Processing

The GAMS program `cdecalib.gms` has a runtime environment variable name `maxthreads`. This variable determines whether the CDE calibration is executed one region at a time or, taking advantage of multi-core processors, several regions at a time. When `threads=1`, the program runs in serial model and solutions listings are generated for each calibration. Otherwise, solution listings are suppressed and multiple (typically 8) demand system calibrations are solved at once.

The grid computing facilities are a relative new and evolving feature in GAMS programming, and they are very useful for applied general equilibrium modeling, so we provide some annotated GAMS code which describes how jobs are submitted and collected:

```
*       When handle(r)=0, region r is inactive, otherwise
*       it is currently being solved:

parameter       handle(r)       Pointer to a grid solve job;

*       When submit(r)=yes, the calibration job for region r
*       is ready to submit.  done(r)=yes when region r is completed.

set             submit(r)       List of regions to submit,
                done(r)         List of regions which are completed;

*       Initialization:

done(r) = no;
handle(r) = 0;

*       Open a file which can be used to update the title bar:

file ktitle;  ktitle.lw=0; ktitle.nd=0;

repeat

          submit(r) = no;

*       Submit as many as maxthreads jobs, choosing from regions
```

```
*       which are neither completed or currently running.

          loop(r$(not (done(r) or handle(r))),

            submit(r) = yes$(card(submit)+card(handle)<maxthreads););

*       Submit all the jobs which have been identified, one at a time,
*       retaining a handle for each of these so we recognize it when
*       it is completed:

          loop(submit(rr),
            rb(r) = yes$sameas(r,rr);
            solve cdecalib using nlp minimizing OBJ;
            handle(rr) = cdecalib.handle;);

*       Update the title bar with a status report of the number of
*       completed jobs, the number remaining and a count of the number
*       currently being processed:

          put_utility 'title'/ktitle 'CDE calibration.'
                ' Finished: ',card(done),','
                ' remaining: ',(card(r)-card(done)),','
                ' running: ',card(handle),'.';

*       Wait for an instance to complete:

          display$ReadyCollect(handle) 'Waiting for next instance to collect';


*       Go through the list of running jobs to find the one which is
*       completed:

          loop(rr$handlecollect(handle(rr)),

*       Unload the solution (lots of assignments suppressed):

            cdelog(rr,"modelstat") = cdecalib.modelstat;
            ...

*       Issue a warning if this handle cannot be deleted:

            display$handledelete(handle(rr)) 'trouble deleting handles' ;

*       Label this region job as inactive and completed:

            handle(rr) = 0;
            done(rr) = yes;);

*       Keep going until we have completed all the jobs or we have run
*       for a long time:

        until (card(done)=card(r)) or timeelapsed > 300;

*       Issue a warning if some jobs did not complete:

        abort$(card(done)<>card(r)) 'CDECALIB jobs did not return:', handle;
```

Integrating CDE Demand in MPSGE

The integration of CDE demand in the MPSGE model is subtle. To begin with, we need to introduce an Armington composite price for final demand. This is done with the `A(i,r)` activity which converts domestic and imported goods into the `PA(i,r)` commodity:

```
$prod:A(i,r)$(rm(r)*vcm(i,r))  s:esubdm(i)
        o:PA(i,r)       q:vcm(i,r)
        i:P(i,r)        q:vdfm(i,"c",r) p:(1+rtfd0(i,"c",r)) a:RA(r) t:rtfd(i,"c",r)
        i:PM(i,r)       q:vifm(i,"c",r) p:(1+rtfi0(i,"c",r)) a:RA(r) t:rtfi(i,"c",r)
```

The CDE demand functions are represented by auxiliary variables, and they enter the MPSGE model as rationing multipliers on regional endowments. The assigned set `cde(r)` is used to indicate whether region r has a CDE demand system.

Here is the tricky business: MPSGE insists that all consumers entering the model demand something. We therefore need to include "phantom" demand and endowment entries which will offset one another in equilibrium. The budget balance condition in the CDE model (the constraint associated with `U(r)` imposes that in equilibrium the value of expenditure for region r (`RA(r)`) equal to the value of the phantom factor endowment vector.

```
$demand:RA(r)$rm(r)  s:0

*       Demand and endowment coefficients for the CD and
*       LES demand systems:

        d:P(cd,r)$(not cde(r))    q:vom(cd,r)
        e:P("sd",r)$(not cde(r))  q:(-vom("sd",r))

*       Coefficient for the CDE demand system:

        e:PA(i,r)$cde(r)        q:(-vcm(i,r))   R:D(i,r)

*       Phantom demand and endowment entries which offset
*       in equilibrium:

        d:PF(f,r)$cde(r)        q:evom(f,r)
        e:PF(f,r)$cde(r)        q:evom(f,r)
...

$constraint:U(r)$(cde(r)*rm(r))
        sum(f, PF(f,r)*evom(f,r)) =e= RA(r);

$constraint:CC(r)$(cde(r)*rm(r))
        sum(i, thetac(i,r) * U(r)**((1-subpar(i,r))*incpar(i,r)) *
                (PA(i,r)/CC(r))**(1-subpar(i,r))) =e= 1;

$constraint:D(i,r)$(rm(r)*vcm(i,r))
        D(i,r) * sum(j$thetac(j,r), thetac(j,r) *
          U(r)**((1-subpar(j,r))*incpar(j,r)) *
                (PA(j,r)/CC(r))**(1-subpar(j,r)))
                   =e= U(r)**((1-subpar(i,r))*incpar(i,r)) *
                        (PA(i,r)/CC(r))**(-subpar(i,r));
```

Appendix D Subdirectories and GAMS Programs

build/		Working directory for dataset construction.
	build.gms	Command script to read, filter and aggregate a set of GTAP datasets. Default configuration: produces datasets gtapingams.gdx, g20.gdx, g20_macro.gdx and g20_iea.gdx for 2011. Calls code/flex2gdx.gms, code/filter.gms, code/gtapaggr.gms and code/cdecalib.gms.
	filterchk.gms	Command script to generate data for Table 6: Filtering results. Calls code/flex2gdx.gms, code/filter.gms. Results are written to a PivotData sheet in filterchk.xlsx.

code/		Code repository – not a working directory.
	flex2gdx.gms	Routine which translates the HAR files from a GTAP .zip archive into GAMS .gdx files. The routine produces an echoprint report of benchmark consistency of the database.
	filter.gms	Filter routine, based on environment parameter –nd, the number of decimal points in the filter.
	gtap9data.gms	Utility routine for read a GTAPINGAMS version 9 dataset.
	gtapaggr.gms	Dataset aggregation routine (call to cdecalib.gms following aggregation must be made by user).
	aggr.gms chktarget.gms checkset.gms	Utility routines called by gtapaggr.gms.

	domain.gms	Standard purpose libinclude routine for extracting the nonzero domain of a parameter, using the mysterios ”`option pd_1<p_1;`” syntax.
	cdecalib.gms	Utility routine to recalibrate CDE demand following dataset aggregation.
	mcp.gms	The canonical static GTAPINGAMS model in algebraic format.
	mge.gms	The canonical static GTAPINGAMS model in tabular MPSGE format.
	gdpcalc.gms	Utility routine for reporting GDP on the basis of income, final demand or sectoral value-added, compatible with either the mcp.gms or mge.gms models. First call declares report parameters.
	loadmdl.gms	Utility routines for switching demand systems (between Cobb-Douglas, LES and CDE) and model closures (GMR versus SOE). The macros included here are described below.

test/		Contains a few routines for verifying consistency of the MGE and MCP models for an aggregated GTAP database.
	run.gms	Command script which calls bmkchk.gms for dataset g20_macro, mcpmge.gms for dataset g20 and cdechk.gms for dataset g20.
	bmkchk.gms	Benchmark consistency is a necessary, but not a sufficient condition that the model is properly specified. This routine checks for the g20_macro dataset in both MGE and MCP formats. This routine produces a replication check for the GMR model and all the single region SOE models.

	mcpmge.gms	Is designed to perform a check of consistency for the MCP and MGE models at a point away from the benchmark equilibrium. It can be used to verify that a solution computed with mge.gms also solves mcp.gms, and that a solution computed with mcp.gms solves mge.gms. The GAMS savepoint and loadpoint commands are used for this purpose.
	cdechk.gms	Verifies that the CDE demand system reproduces the exogenous own-price and income demand elasticities at the benchmark point.

forensics/		Produces some calculations assessing the economic consequences of propotional unilateral changes in trade taxes.
	runs.gms	This GAMS script generates a Windows batch script for sensitivity analysis. The script executes tgrid.gms for several alternative model closures and demand system specifications, and it also executess calls to GDXMERGE which combine the model output in a PivotChart database in Excel workbook results.xlsx.
	runs.bat	Is the batch program written by runs.gms.
	tgrid.gms	Performs a sequence of counterfactual calculations for a single region, with proportional scaling tariff and export subsidy rates to average values between 0 and 10%. These counterfactual calculations are computed with Cobb-Douglas, LES and CDE demand. Macro results (welfare, model status, and global welfare) are returned in parameter results(dsys,tlvl,*). GDP calculations are stored in gdpresults(dsys,tlvl,gdpcat,r,gdpitem).

Macros for model configuation

Two user-callable macros provided in `loadmdl.gms`:

`$macro loaddsys(ff,r)` Loads demand system `ff` for region `r` where `ff` is one of LES, CD or CDE and `r` is a region identifier, either a quoted set element or a subset of the regions `r`. This function alters:

- `les(r)` LES demand system flag for model regions
- `cde(r)` CDE demand system flag
- `esub()` Elasticities of substitution in subsistence and discretionary demand.
- `rtfd(),rtfi()` Tax rates in final demand
- `vdfm(),vifm()` Levels of subsistence and discretionary demand
- `vom()` Output level used to activate aggregate consumer demand, subsistence and discretionary demand.

`$macro loadrm(rr)` Load regions `rr(r)` as endogenous elements of the current model. If `rr` is a set which includes all regions in `r`, then a global multiregional model is produced. If `rr` is singleton or a subset of regions in `r`, then a small open economy closer is provided. This routine alters:

- `rm(r)` Set of regions in the model,
- `rx(r)` Set of regions in rest of world,
- `rnum(r)` Set defining the numeraire region – the region in the model with the largest consumption.
- `vem(i,r)` Exports to ROW regions by regions in the model,
- `rtxs_row(i,r)` Average subsidy rates on exports to ROW regions.
- `rowpfx` Current account balance for the rest of world regions.

The user-callable macros provided in `loadmdl.gms` are invoked as follows in the `tgrid.gms` program:

```
*       Load regions in the model, either a global multiregional
*       model containing all regions or a small open economy model
*       with a single region.

$if not set mdl         $set mdl gmr
$if not set rcalc       $set rcalc chn

$if %mdl%==gmr loadrm(r);
$if %mdl%==soe loadrm("%rcalc%");
```

In the same program, a loop over functional forms is used to calculate scenario results for three different demand systems, as shown below.

GDP reporting

`gdpcalc.gms` declares the following identifiers:[25]

```
set     gdpcat  GDP Categories  /
                expend          Expenditure (C + G + I - (X-M)),
                income          Income (Factor income + taxes),
                valueadded      Sectoral factor earnings plus tax payments,
                total           Total GDP/,

        gdpitem /"X-M",set.g,set.f,
                 revto,revtfd,revtfi,revtf,revtxs,revtms,
                 expend,income,valueadded,chksum/;

alias (gdpitem, gdpi);

parameter       gdp(gdpcat,*,gdpitem)   Real GDP accouting,
                vadd(g, gdpitem, r)     GDP on a value-added basis;
```

GDP reporting in `tgrid.gms` consists of an include statement at the beginning of the program, just after having read `mge.gms`, in a context permitting declarations:

```
*       Read the model and calculate GDP at the benchmark point:

$include %code%mge

$include %code%gdpcalc
```

[25] These names may not be used in the calling program.

The GDP routine is included following scenarios solutions inside the loop over alternative demand systems:

```
loop(dsys,

*       Load the demand system:

  loaddsys(dsys,r);

*       Initialize tax instruments at benchmark values:
  rtms(i,s,r) = rtms0(i,s,r);
  rtxs(i,s,r) = rtxs0(i,s,r);

  loop(tlvl,

*       Assign tax rates for the counterfactual simulation:

    rtms(i,s,rcalc)$max(rtms0(i,s,rcalc),0) = rtms0(i,s,rcalc) * tlvl.val/averate;
    rtxs(i,rcalc,s)$max(-rtxs0(i,rcalc,s),0) = rtxs0(i,rcalc,s) * tlvl.val/averate;

*       Compute the equilibrium values:

$include gtap9.gen
    solve gtap9 using mcp;
    abort$(gtap9.objval>1e-3) "Simulation fails: gtap9.";


*       Store the GDP results for this simulation:

$include %code%gdpcalc

    gdpresults(dsys,tlvl,gdpcat,r,gdpitem) = gdp(gdpcat,r,gdpitem);

...

))
```

Appendix E GAMS Code

Declarations

```
$title   GTAP9inGAMS -- GAMS/MCP Formulation

$if not set yr $set yr 11
$if not set ds $set ds g20_macro
$include "%system.fp%gtap9data"

nonnegative
variables

*$sectors:
        Y(g,r)          Supply
        M(i,r)          Imports
        YT(j)           Transportation services
        FT(f,r)         Specific factor transformation
        X(i,r)          Exports to rest of world (SOE model)
        A(i,r)          Armington final demand quantity index (CDE model)

*$commodities:
        P(g,r)          Domestic output price
        PA(i,r)         Armington final demand price
        PE(g,r)         Export market
        PM(j,r)         Import price
        PT(j)           Transportation services
        PF(f,r)         Primary factors rent
        PS(f,g,r)       Sector-specific primary factors
        PFX             Real exchange rate (SOE model)

*$consumers:
        RA(r)           Representative agent
        ROW             Rest of world (SOE model)

*$auxiliary:
        D(i,r)          CDE Demand (index)
        U(r)            CDE Utility (index)
        CC(r)           CDE Consumption cost index;

equations
        prf_y(g,r)              Supply,
        prf_a(i,r)              Armington demand,
        prf_x(i,r)              Exports,
        prf_m(i,r)              Imports,
        prf_yt(j)               Transportation services,
        prf_ft(f,r)             Factor transformation,

        mkt_p(g,r)              Domestic market,
        mkt_pe(g,r)             Export market,
        mkt_pm(j,r)             Import market,
        mkt_pa(i,r)             Armington market,
        mkt_pt(j)               Transportation service market,
        mkt_pf(f,r)             Primary factor markets,
        mkt_ps(f,j,r)           Specific factor markets,
        mkt_pfx                 Market for foreign exchange,

        inc_ra(r)               Representative agent
        inc_row                 Rest of world agent

        constraint_D            Defines D (CDE model)
        constraint_U            Defines U (CDE model)
        constraint_CC           Defines CC (CDE model);
```

Zero profit (arbitrage) conditions

Sectoral production – Y(g,r)

```
*       Define some macros which diagnose the functional form:

$macro  Leontief(sigma)         (yes$(round(sigma,2)=0))
$macro  CobbDouglas(sigma)      (yes$(round(sigma-1,2)=0))
$macro  CES(sigma)              (yes$(round(sigma-1,2)<>0 and round(sigma,2)<>0))

*       --------------------------------------------------------------------------------
*       Profit function for production and consumption activities:

* $prod:Y(g,r)$ydm(g,r) s:0  m:esub(g,r)  t:etadx(g) i.tl(m):esubd(i)  va:esubva(g)
*       o:P(g,r)        q:(vom(g,r)-vxm(g,r))   a:RA(r)  t:rto(g,r)
*       o:PE(g,r)       q:vxm(g,r)              a:RA(r)  t:rto(g,r)
*       i:P(i,r)        q:vdfm(i,g,r)   p:(1+rtfd0(i,g,r)) i.tl:  a:RA(r) t:rtfd(i,g,r)
*       i:PM(i,r)       q:vifm(i,g,r)   p:(1+rtfi0(i,g,r)) i.tl:  a:RA(r) t:rtfi(i,g,r)
*       i:PS(sf,g,r)    q:vfm(sf,g,r)   p:(1+rtf0(sf,g,r))  va:   a:RA(r) t:rtf(sf,g,r)
*       i:PF(mf,r)      q:vfm(mf,g,r)   p:(1+rtf0(mf,g,r))  va:   a:RA(r) t:rtf(mf,g,r)

* $prod:Y(g,r)$(rm(r)*vom(g,r)) s:0 m:esub(g,r) t:etadx(g) i.tl(m):esubdm(i)  va(s):esubva(g)
*       o:P(g,r)        q:(vom(g,r)-vxm(g,r))   a:RA(r)  t:rto(g,r)
*       o:PE(g,r)       q:vxm(g,r)              a:RA(r)  t:rto(g,r)
*       i:P(i,r)        q:vdfm(i,g,r)   p:(1+rtfd0(i,g,r)) i.tl:  a:RA(r) t:rtfd(i,g,r)
*       i:PM(i,r)       q:vifm(i,g,r)   p:(1+rtfi0(i,g,r)) i.tl:  a:RA(r) t:rtfi(i,g,r)
*       i:PS(sf,g,r)    q:vfm(sf,g,r)   p:(1+rtf0(sf,g,r))  va:   a:RA(r) t:rtf(sf,g,r)
*       i:PF(mf,r)      q:vfm(mf,g,r)   p:(1+rtf0(mf,g,r))  va:   a:RA(r) t:rtf(mf,g,r)

*       Benchmark value shares:

parameter
        cf0(g,r)                Factor cost
        cm0(g,r)                Material cost
        cy0(g,r)                Reference total cost,
        theta_f(g,r)            Value added share of cost,
        thetaf(f,g,r)           Factor share of value added,
        thetam(i,g,r)           Armington share of material cost,
        thetad(i,g,r)           Domestic share of Armington composite,
        theta_e(g,r)            Export share of output,
        thetayt(j,r)            Value share,
        thetavxmd(i,s,r)        Value share of goods in imports,
        thetavtwr(j,i,s,r)      Value share of transportation services,
        theta_m(i,s,r)          Bilateral import value share
        vxmt(i,s,r)             Value of imports gross transport cost,
        vxmd0(i,r,s)            Trade - bilateral exports at market prices (dataset values),
        vtwr0(j,i,r,s)          Trade - margins for international transportation at world prices,
        thetavfm(f,j,r)         Value shares of specific factors;

vxmd0(i,r,s)  = vxmd(i,r,s);
vtwr0(j,i,r,s) = vtwr(j,i,r,s);

$setlocal datetime %system.date%%system.time%
$echo   *       Calibration code for mcp.gms (%datetime%)    >gtap9.gen

*       Save copy of the benchmark trade flows so that we can turn off
*       demand as needed in the SOE model:

vxmd(i,r,s)  = vxmd0(i,r,s);
vxmd(i,rx(r),rm(s)) = vxmd0(i,r,s)$vem(i,r);

vtwr(j,i,r,s)  = vtwr0(j,i,r,s);
vtwr(j,i,rx(r),rm(s)) = vtwr0(j,i,r,s)$vem(i,r);

*       User cost indices for factors, domestic and imported
*       intermediate inputs:

$macro P_PF(f,g,r)  (((PF(f,r)$mf(f)+PS(f,g,r)$sf(f))*(1+rtf(f,g,r)) \
                / (1+rtf0(f,g,r)))$thetaf(f,g,r)  + 1$(thetaf(f,g,r)=0))
```

```
$macro P_D(i,g,r)  ((P(i,r)*(1+rtfd(i,g,r)) \
               / (1+rtfd0(i,g,r)))$thetad(i,g,r) + 1$(not thetad(i,g,r)))

$macro P_I(i,g,r)  ((PM(i,r)*(1+rtfi(i,g,r)) \
               / (1+rtfi0(i,g,r)))$(1-thetad(i,g,r)) + 1$(thetad(i,g,r)=1))

*       Compensated cost functions:

$if defined f_ $abort "The CF(g,r) macro requires a uniquely defined alias for f."
alias (f,f_);

*       Factor cost index:

$macro C_F(g,r) ( \
        ( (sum(f_$thetaf(f_,g,r),  thetaf(f_,g,r)*P_PF(f_,g,r)))$Leontief(esubva(g)) + \
          (prod(f_$thetaf(f_,g,r), P_PF(f_,g,r)**thetaf(f_,g,r)))$CobbDouglas(esubva(g)) + \
          (sum(f_$thetaf(f_,g,r), \
              thetaf(f_,g,r)*P_PF(f_,g,r)**(1-esubva(g)) \
          )**(1/(1-esubva(g))))$CES(esubva(g)))$cf0(g,r) )

*       Armington cost index:

$macro C_A(i,g,r) ( \
        (thetad(i,g,r)*P_D(i,g,r) + (1-thetad(i,g,r))*P_I(i,g,r))$Leontief(esubdm(i)) + \
        (P_D(i,g,r)**thetad(i,g,r) * P_I(i,g,r)**(1-thetad(i,g,r)))$CobbDouglas(esubdm(i)) + \
        ( (thetad(i,g,r) *P_D(i,g,r)**(1-esubdm(i)) + \
        (1-thetad(i,g,r))*P_I(i,g,r)**(1-esubdm(i)))**(1/(1-esubdm(i))))$CES(esubdm(i)))

$if defined i_ $abort "The C_M(g,r) macro requires a uniquely defined alias for i."
alias (i,i_);
$macro C_M(g,r) ( \
 ( (sum(i_$thetam(i_,g,r),  thetam(i_,g,r) * \
         C_A(i_,g,r)**(1-esub(g,r))))**(1/(1-esub(g,r))))$CES(esub(g,r)) + \
    prod(i_$thetam(i_,g,r), C_A(i_,g,r)**thetam(i_,g,r))$CobbDouglas(esub(g,r)) + \
    sum(i_$thetam(i_,g,r),  thetam(i_,g,r) * C_A(i_,g,r))$Leontief(esub(g,r)) ) )

*       Unit cost function:

$macro C_Y(g,r) (theta_f(g,r)*C_F(g,r) + (1-theta_f(g,r))*C_M(g,r))

*       Unit revenue function:

$macro R_Y(g,r) (  P(g,r)$(not vxm(g,r)) + \
 (((1-theta_e(g,r))*  P(g,r)**(1+etadx(g)) + \
      theta_e(g,r) * PE(g,r)**(1+etadx(g)))**(1/(1+etadx(g))))$vxm(g,r) )

*       Demand functions:

$macro DDY(i,g,r)       ((vdfm(i,g,r)*Y(g,r)*(C_A(i,g,r)/P_D(i,g,r))**esubdm(i) * \
                                    (C_M(g,r)/C_A(i,g,r))**esub(g,r))$vdfm(i,g,r))

$macro DDA(i,r)         ((vdfm(i,"C",r)*A(i,r)*(C_A(i,"C",r)/P_D(i,"C",r))**esubdm(i) * \
                                      (C_M("C",r)/C_A(i,"C",r))**esub("c",r))$vdfm(i,"c",r))

$macro DIY(i,g,r)       ((vifm(i,g,r)    * Y(g,r) * (C_A(i,g,r) /P_I(i,g,r)   )**esubdm(i) * \
                                        (C_M(g,r)/C_A(i,g,r))**esub(g,r))$vifm(i,g,r))

$macro DIA(i,r)         ((vifm(i,"c",r) * A(i,r) * \
                          (C_A(i,"C",r)/P_I(i,"C",r))**esubdm(i))$(vifm(i,"C",r)*vcm(i,r)))

$macro DDFM(i,g,r)      (DDY(i,g,r)$vom(g,r) + DDA(i,r)$(sameas(g,"c") and vcm(i,r)))

$macro DIFM(i,g,r)      (DIY(i,g,r)$vom(g,r) + DIA(i,r)$(sameas(g,"c") and vcm(i,r)))

$macro DFM(f,g,r)       (vfm(f,g,r)   * Y(g,r) * (C_F(g,r)/P_PF(f,g,r))**esubva(g))$vfm(f,g,r)

$macro S_D(g,r)         ((1$(not vxm(g,r)) + ((P(g,r) /R_Y(g,r))**etadx(g))$vxm(g,r)) * \
                        (vom(g,r)-vxm(g,r)) * Y(g,r))

$macro S_E(g,r)         (vxm(g,r)*Y(g,r)*(PE(g,r)/R_Y(g,r))**etadx(g))
```

```
*       Associated tax revenue flows:

$macro REV_TO(g,r)  (rto(g,r)*R_Y(g,r)*vom(g,r)*Y(g,r))

$macro REV_TFD(i,g,r)   ((rtfd(i,g,r) * P(i,r)  * DDFM(i,g,r))$(rm(r)*rtfd(i,g,r)*vdfm(i,g,r)))

$macro REV_TFI(i,g,r)   ((rtfi(i,g,r) * PM(i,r) * DIFM(i,g,r))$(rm(r)*rtfi(i,g,r)*vifm(i,g,r)))

$macro REV_TF(f,g,r)    ((rtf(f,g,r)  * DFM(f,g,r) * \
        ((PS(f,g,r))$sf(f)+(PF(f,r))$mf(f)))$(rm(r)*vom(g,r)*rtf(f,g,r)*vfm(f,g,r)))

prf_y(g,r)$(rm(r)*vom(g,r))..   cy0(g,r)*C_Y(g,r)  =e= vom(g,r)*(1-rto(g,r))*R_Y(g,r);
```

International transportation services – `YT(j)`

```
*       $prod:YT(j)$vtw(j)  s:1
*               o:PT(j)                         q:vtw(j)
*               i:PE(j,r)$(rm(r)*vxm(j,r))      q:vst(j,r)
*               i:P(j,r)$(rm(r)*(not vxm(j,r))) q:vst(j,r)
*               i:PFX                           q:(sum(rx(r),vst(j,r)))


$macro PYT(j,r)  (PX(j,r)$rm(r) + (PFX)$rx(r))

prf_yt(j)$vtw(j).. sum(r,vst(j,r)) *
        prod(r, PYT(j,r)**thetayt(j,r)) =e= vtw(j)*PT(j);


*       Demand Function:

$macro DST(j,r)    ((vst(j,r)*YT(j)*PT(j)/PYT(j,r))$vst(j,r))
```

Aggregate imports – `M(i,r)`

```
* $prod:M(i,r)$(rm(r)*vim(i,r)) s:esubm(i)  s.tl:0
*       o:PM(i,r)       q:vim(i,r)
*       i:PE(i,s)$(rm(s)*vxm(i,s))      q:vxmd(i,s,r)   p:pvxmd(i,s,r) s.tl:
* +             a:RA(s) t:(-rtxs(i,s,r)) a:RA(r) t:(rtms(i,s,r)*(1-rtxs(i,s,r)))
*       i:P(i,s)$(rm(s)*(not vxm(i,s))) q:vxmd(i,s,r)   p:pvxmd(i,s,r) s.tl:
* +             a:RA(s) t:(-rtxs(i,s,r)) a:RA(r) t:(rtms(i,s,r)*(1-rtxs(i,s,r)))
*       i:PT(j)#(s)                     q:vtwr(j,i,s,r) p:pvtwr(i,s,r) s.tl:
* +             a:RA(r) t:rtms(i,s,r)
*       i:PE(i,rx)$vem(i,rx)            q:vxmd(i,rx,r)  p:pvxmd(i,rx,r) rx.tl:
* +             a:ROW t:(-rtxs(i,rx,r))     a:RA(r) t:(rtms(i,rx,r)*(1-rtxs(i,rx,r)))

*       User cost indices:

$macro P_M(i,s,r)    ((PX(i,s) * \
        (1-rtxs(i,s,r))*(1+rtms(i,s,r))/pvxmd(i,s,r))$vxmd(i,s,r)  + 1$(vxmd(i,s,r)=0))

$macro P_T(j,i,s,r)  ((PT(j)*(1+rtms(i,s,r))/pvtwr(i,s,r))$vtwr(j,i,s,r)+1$(vtwr(j,i,s,r)=0))

*       Price index of bilateral imports (Leontief cost function):

$if defined j1 $abort "The PT_M(i,s,r) macro requires a uniquely defined alias for j."
alias (j,j1);
$macro PT_M(i,s,r)  (P_M(i,s,r)*thetavxmd(i,s,r) + sum(j1, P_T(j1,i,s,r)*thetavtwr(j1,i,s,r)))

*       Unit cost function for imports (CES):

$if defined s_ $abort "The CIM(i,r) macro requires a uniquely defined alias for s."
alias (s,s_);
$macro CIM(i,r)  ( \
   sum(s_, theta_m(i,s_,r) * PT_M(i,s_,r) )$Leontief(esubm(i)) + \
   prod(s_, PT_M(i,s_,r)**theta_m(i,s_,r) )$CobbDouglas(esubm(i)) + \
```

```
  (sum(s_, theta_m(i,s_,r) * PT_M(i,s_,r)**(1-esubm(i)))**(1/(1-esubm(i))))$CES(esubm(i)) )

prf_m(i,rm(r))$vim(i,r)..       CIM(i,r)*vim(i,r) =e= PM(i,r)*vim(i,r);

*       Demand function:

$macro DXMD(i,s,r)    ((vxmd(i,s,r)    * M(i,r) * (PM(i,r)/PT_M(i,s,r))**esubm(i))$vxmd(i,s,r))
$macro DTWR(j,i,s,r)  ((vtwr(j,i,s,r) * M(i,r) * (PM(i,r)/PT_M(i,s,r))**esubm(i))$vtwr(j,i,s,r))

*       Associated tax revenue:

alias (rm,sm);

$macro XMD(i,s,r)  ((DXMD(i,s,r)/vxmd(i,s,r))$(rm(r)*vim(i,r)*vxmd(i,s,r)) \
                + X(i,s)$(rm(s) and rx(r) and vxmd(i,s,r)))
$macro REV_TXS(i,s,r)  ((XMD(i,s,r)*vxmd(i,s,r)*PX(i,s)*rtxs(i,s,r))$(rm(s) and vxmd(i,s,r)))

$if defined j2 $abort "The REVTMS(r) macro requires a uniquely defined alias for j."
alias (j,j2);

$macro REV_TMS(i,s,r)    ((XMD(i,s,r)*rtms(i,s,r)* \
                ((1-rtxs(i,s,r))*vxmd(i,s,r)*PX(i,s) + \
                 sum(j$vtwr(j,i,s,r), PT(j)*vtwr(j,i,s,r))))$(rm(r)*vxmd(i,s,r)*rtms(i,s,r)))
```

Sector-specific factor transformation – `FT(f,r)`

```
*       $prod:FT(sf,r)$evom(sf,r)  t:etrae(sf)
*               o:PS(sf,j,r)     q:vfm(sf,j,r)
*               i:PF(sf,r)       q:evom(sf,r)

$if defined j3 $abort "The PVFM(sf,r) macro requires a uniquely defined alias for j."
alias (j,j3);
$macro PVFM(sf,r)  (sum(j3,thetavfm(sf,j3,r)*PS(sf,j3,r)**(1+etrae(sf)))**(1/(1+etrae(sf))))

prf_ft(sf,rm(r))$evom(sf,r)..   evom(sf,r)*PF(sf,r) =e= evom(sf,r)*PVFM(sf,r);
```

Market clearance

These equations do not appear explicitly in the MPSGE model, as they are generated automatically on the basis of the production function information provided above.

Domestic production and final demand – `PY(g,r)`

```
alias (g,gg);

mkt_p(g,rm(r))$(rm(r)*(vom(g,r)-vxm(g,r)))..
        S_D(g,r) =e= (RA(r)/P(g,r))$(vom(g,r) and cd(g)) +
                vom(g,r)$(sameas(g,"G") or sameas(g,"I") or sameas(g,"sd")) +
                sum(i$sameas(i,g),
                  sum(gg,DDFM(i,gg,r)) +
                        (X(i,r)*vem(i,r)+sum(sm(s),DXMD(i,r,s)) + DST(i,r))$(not vxm(i,r)));
```

Export supply and demand – `PE(i,r)`

```
mkt_pe(i,r)$(rm(r)*vxm(i,r) or rx(r)*vem(i,r))..
        S_E(i,r)$rm(r) + (X(i,r)*vem(i,r))$rx(r) =e=

                sum(sm(s),DXMD(i,r,s)) + (X(i,r)*vem(i,r) + DST(i,r))$rm(r);
```

Composite imports – `PM(i,r)`

```
mkt_pm(i,rm(r))$vim(i,r)..      M(i,r) * vim(i,r) =e= sum(g, DIFM(i,g,r));
```

Transport services – `PT(j)`

```
mkt_pt(j)$vtw(j)..   YT(j) * vtw(j) =e= sum((i,s,r)$vtwr(j,i,s,r),
                                  DTWR(j,i,s,r)$rm(r) + vtwr(j,i,s,r)$rx(r));
```

Primary factors – `PF(f,r)`

```
mkt_pf(f,rm(r))$evom(f,r)..
        evom(f,r) =e= sum(j, DFM(f,j,r))$mf(f) + (evom(f,r)*FT(f,r))$sf(f);
```

Specific factors – `PS(f,j,r)`

```
mkt_ps(sf,j,rm(r))$vfm(sf,j,r)..
        vfm(sf,j,r) * (PS(sf,j,r)/PF(sf,r))**etrae(sf) * FT(sf,r) =e= DFM(sf,j,r);
```

Regional income balance – `RA(r)`

```
* $demand:RA(r)$rm(r)  s:0
*       d:P("c",r)              q:vom("c",r)
*       e:P("g",r)              q:(-vom("g",r))
*       e:P("i",r)              q:(-vom("i",r))
*       e:P("i",rnum)$rm(rnum)  q:vb(r)
*       e:PFX$rx(rnum)          q:vb(r)
*       e:PF(f,r)               q:evom(f,r)
*       d:PF(f,r)$cde(r)        q:evom(f,r)
*       e:PF(f,r)$cde(r)        q:evom(f,r)
*       e:PA(i,r)               q:(-vcm(i,r))   R:D(i,r)

inc_ra(rm(r))$(RA.LO(r) < RA.UP(r))..
        RA(r) =e=       ( sum(f$evom(f,r), PF(f,r)*evom(f,r))
                        - sum(i$vcm(i,r), PA(i,r)*vcm(i,r)*D(i,r)) )$cde(r)
                        + vb(r)*((PFX)$rx(rnum)+P("i",rnum)$rm(rnum))
                        - P("g",r)*vom("g",r)
                        - P("i",r)*vom("i",r)
                        - (P("sd",r)*vom("sd",r))$vom("sd",r)
                        + sum(f$evom(f,r), PF(f,r)*evom(f,r))
                        + sum(g,REV_TO(g,r))
                        + sum((i,g),REV_TFD(i,g,r))
                        + sum((i,g),REV_TFI(i,g,r))
                        + sum((f,g),REV_TF(f,g,r))
                        - sum((i,s),REV_TXS(i,r,s))
                        + sum((i,s),REV_TMS(i,s,r));
```

Model Declaration

```
*       Two flavors:  The MCP declaration declares equations
*       and associates these with variables.  The CNS declaration simply declares
*       equations:

model gtap9 /
        prf_y.Y,prf_x.X,prf_a.A,prf_m.M,prf_yt.YT,prf_ft.FT,
```

```
        mkt_p.P,mkt_pe.PE,mkt_pa.PA,mkt_pm.PM,mkt_pt.PT,mkt_pf.PF,mkt_ps.PS,mkt_pfx.PFX,
        constraint_D.D, constraint_U.U, constraint_CC.CC,
        inc_ra.RA, inc_row.ROW/;

model gtap9cns /
        prf_y,prf_x,prf_a,prf_m,prf_yt,prf_ft,
        mkt_p,mkt_pe,mkt_pa,mkt_pm,mkt_pt,mkt_pf,mkt_ps,mkt_pfx,
        constraint_D, constraint_U, constraint_CC,
        inc_ra, inc_row/;

*       Assign default values:

Y.L(g,r) = 1;
A.L(i,r) = 1;
X.L(i,r) = 1;
M.L(i,r) = 1;
YT.L(j) = 1;
FT.L(sf,r) = 1;
P.L(g,r) = 1;
PA.L(i,r) = 1;
PM.L(j,r) = 1;
PM.FX(j,r)$(not vim(j,r))  = 0;
PT.L(j) = 1;
PF.L(f,r) = 1;
PS.L(sf,j,r) = 1;
PFX.L = 1;
X.L(i,r)$vem(i,r) = 1;
PE.L(i,r)$vxm(i,r) = 1;
```

Macros to Move Between Closures and Demand Systems

```
*       Declare some macros to move between model types:

$macro loaddsys(ff,r) \
        if (sameas(ff,"les"), loadles(r); ); \
        if (sameas(ff,"cd"),   loadcd(r); ); \
        if (sameas(ff,"cde"), loadcde(r); );

$macro loadles(r)  \
        loadcd(r); \
        les(r) = yes; \
        esub("sd",r) = 0; \
        esub("dd",r) = 1; \
        rtfd(i,lesd,r) = rtfd(i,"c",r); \
        rtfi(i,lesd,r) = rtfi(i,"c",r); \
        vdfm(i,"sd",r) = vdfm(i,"c",r)*(1-betales(r)*eta(i,r)); \
        vifm(i,"sd",r) = vifm(i,"c",r)*(1-betales(r)*eta(i,r)); \
        vdfm(i,"dd",r) = vdfm(i,"c",r)*betales(r)*eta(i,r); \
        vifm(i,"dd",r) = vifm(i,"c",r)*betales(r)*eta(i,r); \
        vom(lesd,r) = sum(i,vdfm(i,lesd,r)*(1+rtfd(i,lesd,r)) +  \
                    vifm(i,lesd,r)*(1+rtfi(i,lesd,r))); \
        vom("c",r) = 0;

$macro  loadcde(r) \
        loadcd(r); \
        cde(r) = yes; \
        vcm(i,r) = vdfm(i,"c",r)*(1+rtfd(i,"c",r)) + vifm(i,"c",r)*(1+rtfi(i,"c",r)); \
        vom(cd,r) = 0; \
        U.L(r) = 1; \
        D.L(i,r)$vcm(i,r) = 1; \
        CC.L(r) = 1;

$macro loadcd(r)  \
        unloadcde(r) \
        unloadles(r)

$macro unloadles(r)  \
        rtfd(i,lesd,r) = 0; \
        rtfi(i,lesd,r) = 0; \
        vdfm(i,lesd,r) = 0; \
        vifm(i,lesd,r) = 0; \
        vom("c",les(r)) = sum(i,vdfm(i,"c",r)*(1+rtfd(i,"c",r)) +  \
                    vifm(i,"c",r)*(1+rtfi(i,"c",r))); \
        vom(lesd,les(r)) = 0; \
        les(r) = no;

$macro unloadcde(r) \
        vom("c",cde(r)) = sum(i,vdfm(i,"c",r)*(1+rtfd(i,"c",r)) +  \
                    vifm(i,"c",r)*(1+rtfi(i,"c",r))); \
        vcm(i,cde(r)) = 0; \
        U.L(cde(r)) = 0; \
        D.L(i,cde(r)) = 0; \
        CC.L(cde(r)) = 0; \
        cde(r) = no;

$if defined ii_ $abort "The loadrm(r) macro requires a uniquely defined alias for i."
$if defined ss_ $abort "The loadrm(r) macro requires a uniquely defined alias for s."

alias (r,r_,rr), (j,j_),(i,ii_), (s,ss_), (f,f_);

$macro loadrm(r) \
        rm(r_) = no; \
        rm(r) = yes; \
        rnum(r) = yes$(rm(r) and (vom("c",r)+vom("sd",r)+vom("dd",r)= \
                  smax(rm,vom("c",rm)+vom("sd",rm)+vom("dd",rm)))); \
        rx(r_) = (not rm(r_)); \
        vem(ii_,r_) = sum(rx,vxmd(ii_,r_,rx))$rm(r_) + sum(rm,vxmd(ii_,r_,rm))$rx(r_); \
        vem(ii_,r_)$(not round(vem(ii_,r_),5)) = 0; \
        rtxs_row(ii_,r_) = \
          (sum(rx,vxmd(ii_,r_,rx)*rtxs(ii_,r_,rx))/vem(ii_,r_))$(rm(r_) and vem(ii_,r_));\
```

```
rowpfx = sum(rx,  vom("c",rx)+vom("sd",rx)+vom("dd",rx)+ \
         sum(ii_,vcm(ii_,rx))+sum((j_,ii_,ss_),vtwr(j_,ii_,ss_,rx))) + sum(rm,vb(rm));\
pem0(ii_,r_) = (1-rtxs_row(ii_,r_))$rm(r_);
```

GDP Reporting Code

```
$stitle         GAMS Code for GDP Reporting from the MGE or MCP Model

$ifthen.undefined not defined gdpcat

*       Declaration:

        set     gdpcat /expend,income,valueadded, total/,

                gdpitem /"X-M",set.g,set.f,
                         revto,revtfd,revtfi,revtf,revtxs,revtms,
                         expend,income,valueadded,chksum/;

        alias (gdpitem, gdpi);

        parameter       gdp(gdpcat,*,gdpitem)   Real GDP accouting
                        vadd(g, gdpitem, r)     GDP on a value-added basis;

$endif.undefined


$ondotl
loop(rm(r),
        gdp("expend",r,"C") =  RA(r)/pnum
               + ( sum(i$vcm(i,r), PA(i,r)*vcm(i,r)*D(i,r))/pnum
                 - sum(f_$evom(f_,r), PF(f_,r)*evom(f_,r)) /pnum )$cde(r)
               + (P("sd",r)*vom("sd",r)/pnum                    )$vom("sd",r);

        gdp("expend",r,"X-M") = -vb(r)*(PFX$rx(rnum)+P("i",rnum)$rm(rnum))/pnum;
        gdp("expend",r,"g") = P("g",r)*vom("g",r)/pnum ;
        gdp("expend",r,"i") = P("i",r)*vom("i",r)/pnum ;

        vadd(i,gdpitem(f),r) = DFM(f,i,r)*(PF.L(f,r)*mf(f)+PS.L(f,i,r)$sf(f))/pnum;
        vadd(g,"revto",r) = REV_TO(g,r)/pnum;
        vadd(g,"revtfd",r) = sum(i,REV_TFD(i,g,r))/pnum;
        vadd(g,"revtfi",r) = sum(i,REV_TFI(i,g,r))/pnum;
        vadd(g,"revtf",r) = sum(f,REV_TF(f,g,r))/pnum;
        vadd(i,"revtxs",r) = -sum(s,REV_TXS(i,r,s))/pnum;
        vadd(i,"revtms",r) = sum(s, REV_TMS(i,s,r))/pnum;

        gdp("valueadded",r,gdpitem(g)) = sum(gdpi,vadd(g,gdpi,r));

        gdp("income",r,gdpitem(f)) = PF(f,r)*evom(f,r)/pnum;
        gdp("income",r,"revto") =  sum(g,REV_TO(g,r))/pnum;
        gdp("income",r,"revtfd") = sum((i,g), REV_TFD(i,g,r))/pnum;
        gdp("income",r,"revtfi") = sum((i,g), REV_TFI(i,g,r))/pnum;
        gdp("income",r,"revtf")  = sum((f,g), REV_TF(f,g,r))/pnum;
        gdp("income",r,"revtxs") = -sum((i,s), REV_TXS(i,r,s))/pnum;
        gdp("income",r,"revtms") = sum((i,s), REV_TMS(i,s,r))/pnum;

        gdp("total",r,"valueadded") = sum((g,gdpitem),vadd(g,gdpitem,r));

        gdp("total",r,"expend") =
                        gdp("expend",r,"c")
                        + gdp("expend",r,"i")
                        + gdp("expend",r,"g")
                        + gdp("expend",r,"x-m");
        gdp("total",r,"income") =
                        sum(gdpitem(f), gdp("income",r,gdpitem))
                        + gdp("income",r,"revto")
                        + gdp("income",r,"revtfd")
                        + gdp("income",r,"revtfi")
                        + gdp("income",r,"revtf")
```

```
                        + gdp("income",r,"revtxs")
                        + gdp("income",r,"revtms");
        gdp("total",r,"chksum") =
                abs(gdp("total",r,"expend") - gdp("total",r,"income")) +
                abs(gdp("total",r,"expend") - gdp("total",r,"valueadded")) ;
);
```

GAMS/MPSGE Version

```
$title   GTAP9inGAMS -- GAMS/MPSGE Formulation

$if not set yr $set yr 11
$if not set ds $set ds g20
$include "%system.fp%gtap9data"

$ontext
$model:gtap9

$sectors:
        Y(g,r)$(rm(r)*vom(g,r))          ! Supply
        M(i,r)$(rm(r)*vim(i,r))          ! Imports
        YT(j)$vtw(j)                     ! Transportation services
        FT(f,r)$(rm(r)*sf(f)*evom(f,r))  ! Specific factor transformation
        A(i,r)$(rm(r)*vcm(i,r))          ! Armington final demand quantity index (CDE model)
        X(i,r)$vem(i,r)                  ! Exports to or from rest of world (SOE model)

$commodities:
        P(g,r)$(rm(r)*(vom(g,r)-vxm(g,r)))              ! Domestic output price
        PE(g,r)$(rx(r)*vem(g,r)+rm(r)*vxm(g,r))         ! Export market
        PM(j,r)$(rm(r)*vim(j,r))                        ! Import price
        PT(j)$vtw(j)                                    ! Transportation services
        PF(f,r)$(rm(r)*evom(f,r))                       ! Primary factors rent
        PS(f,g,r)$(rm(r)*sf(f)*vfm(f,g,r))              ! Sector-specific primary factors
        PFX$card(rx)                                    ! Real exchange rate (SOE model)
        PA(i,r)$(rm(r)*vcm(i,r))                        ! Armington final demand price

$consumers:
        RA(r)$rm(r)             ! Representative agent
        ROW$card(rx)            ! Rest of world (SOE model)

$auxiliary:
        D(i,r)$(rm(r)*vcm(i,r)) ! Demand (index)
        U(r)$(rm(r)*cde(r))     ! Utility (index)
        CC(r)$(rm(r)*cde(r))    ! Consumption cost index

$prod:Y(g,r)$(rm(r)*vom(g,r)) s:0 t:etadx(g) m:esub(g,r) va:esubva(g) i.tl(m):esubdm(i)
        o:P(g,r)        q:(vom(g,r)-vxm(g,r))    a:RA(r)  t:rto(g,r)      p:(1-rto(g,r))
        o:PE(g,r)       q:vxm(g,r)               a:RA(r)  t:rto(g,r)      p:(1-rto(g,r))
        i:P(i,r)        q:vdfm(i,g,r)    p:(1+rtfd0(i,g,r)) i.tl:  a:RA(r) t:rtfd(i,g,r)
        i:PM(i,r)       q:vifm(i,g,r)    p:(1+rtfi0(i,g,r)) i.tl:  a:RA(r) t:rtfi(i,g,r)
        i:PS(sf,g,r)    q:vfm(sf,g,r)    p:(1+rtf0(sf,g,r))  va:   a:RA(r) t:rtf(sf,g,r)
        i:PF(mf,r)      q:vfm(mf,g,r)    p:(1+rtf0(mf,g,r))  va:   a:RA(r) t:rtf(mf,g,r)

$prod:YT(j)$vtw(j)  s:1
        o:PT(j)                         q:vtw(j)
        i:PE(j,r)$(rm(r)*vxm(j,r))      q:vst(j,r)
        i:P(j,r)$(rm(r)*(not vxm(j,r))) q:vst(j,r)
        i:PFX                           q:(sum(rx,vst(j,rx)))

$prod:M(i,r)$(rm(r)*vim(i,r))   s:esubm(i)  s.tl:0
        o:PM(i,r)                       q:vim(i,r)
        i:PE(i,rm)$vxm(i,rm)            q:vxmd(i,rm,r)  p:pvxmd(i,rm,r) rm.tl:
+               a:RA(rm) t:(-rtxs(i,rm,r)) a:RA(r) t:(rtms(i,rm,r)*(1-rtxs(i,rm,r)))
        i:P(i,rm)$(not vxm(i,rm))       q:vxmd(i,rm,r)  p:pvxmd(i,rm,r) rm.tl:
+               a:RA(rm) t:(-rtxs(i,rm,r)) a:RA(r) t:(rtms(i,rm,r)*(1-rtxs(i,rm,r)))
        i:PT(j)#(s)                     q:vtwr(j,i,s,r) p:pvtwr(i,s,r) s.tl: a:RA(r) t:rtms(i,s,r)
        i:PE(i,rx)$vem(i,rx)            q:vxmd(i,rx,r)  p:pvxmd(i,rx,r) rx.tl:
+               a:ROW    t:(-rtxs(i,rx,r)) a:RA(r) t:(rtms(i,rx,r)*(1-rtxs(i,rx,r)))

$prod:FT(sf,r)$(rm(r)*evom(sf,r))  t:etrae(sf)
        o:PS(sf,j,r)    q:vfm(sf,j,r)
        i:PF(sf,r)      q:evom(sf,r)

$demand:RA(r)$rm(r)  s:0
        d:P(cd,r)               q:vom(cd,r)
        e:P("g",r)              q:(-vom("g",r))
        e:P("i",r)              q:(-vom("i",r))
```

```
        e:P("i",rnum)$rm(rnum)  q:vb(r)
        e:PFX$rx(rnum)          q:vb(r)
        e:PF(f,r)               q:evom(f,r)

*       Coefficient for the LES demand system:

        e:P("sd",r)             q:(-vom("sd",r))

*       Coefficient for the CDE demand system:

        d:PF(f,r)$cde(r)        q:evom(f,r)
        e:PF(f,r)$cde(r)        q:evom(f,r)
        e:PA(i,r)$cde(r)        q:(-vcm(i,r))   R:D(i,r)

*       -----------------------------------------------------------------
*       Additional code for the SOE closure:

$prod:X(i,r)$(rm(r)*vem(i,r))
        o:PFX                           q:(pem0(i,r)*vem(i,r))
        i:P(i,r)$(not vxm(i,r))         q:vem(i,r)      a:RA(r) t:(-rtxs_row(i,r))
        i:PE(i,r)$vxm(i,r)              q:vem(i,r)      a:RA(r) t:(-rtxs_row(i,r))

$prod:X(i,r)$(rx(r)*vem(i,r))
        o:PE(i,r)               q:vem(i,r)
        i:PFX                   q:vem(i,r)

$demand:ROW$card(rx)
        e:PFX                   q:rowpfx
        e:P("i",rnum)$rm(rnum)  q:(-sum(rm,vb(rm)))
        e:PT(j)                 q:(-sum((i,s,rx), vtwr(j,i,s,rx)))
        d:PFX

$prod:A(i,r)$(rm(r)*vcm(i,r))  s:esubdm(i)
        o:PA(i,r)       q:vcm(i,r)
        i:P(i,r)        q:vdfm(i,"c",r) p:(1+rtfd0(i,"c",r)) a:RA(r) t:rtfd(i,"c",r)
        i:PM(i,r)       q:vifm(i,"c",r) p:(1+rtfi0(i,"c",r)) a:RA(r) t:rtfi(i,"c",r)


$constraint:D(i,r)$(rm(r)*vcm(i,r))
        D(i,r) * sum(j$thetac(j,r), thetac(j,r) *
          U(r)**((1-subpar(j,r))*incpar(j,r)) *
                (PA(j,r)/CC(r))**(1-subpar(j,r)))
                   =e= U(r)**((1-subpar(i,r))*incpar(i,r)) *
                        (PA(i,r)/CC(r))**(-subpar(i,r));

$constraint:U(r)$(cde(r)*rm(r))
        sum(f, PF(f,r)*evom(f,r)) =e= RA(r);

$constraint:CC(r)$(cde(r)*rm(r))
        sum(i, thetac(i,r) * U(r)**((1-subpar(i,r))*incpar(i,r)) *
                (PA(i,r)/CC(r))**(1-subpar(i,r))) =e= 1;

$report:
        v:V_VST(i,r)$(vst(i,r) and rm(r) and (not vxm(i,r)))    i:P(i,r)        prod:YT(i)
        v:V_VST(i,r)$(vst(i,r) and rm(r) and vxm(i,r))          i:PE(i,r)       prod:YT(i)
        v:V_VDM(g,r)$(rm(r)*(vom(g,r)-vxm(g,r)))                o:P(g,r)        prod:Y(g,r)
        v:V_XMD(i,s,r)$(vxmd(i,s,r)*rm(s)*vxm(i,s))             i:PE(i,s)       prod:M(i,r)
        v:V_XMD(i,s,r)$(vxmd(i,s,r)*rx(s))                      i:PE(i,s)       prod:M(i,r)
        v:V_VXM(g,r)$(rm(r)*vom(g,r)*vxm(g,r))                  o:PE(g,r)       prod:Y(g,r)
        v:V_DFM(i,g,r)$(rm(r)*vom(g,r)*vdfm(i,g,r))             i:P(i,r)        prod:Y(g,r)
        v:V_IFM(i,g,r)$(rm(r)*vom(g,r)*vifm(i,g,r))             i:PM(i,r)       prod:Y(g,r)
        v:V_FM(f,g,r)$(sf(f)*rm(r)*vom(g,r)*vfm(f,g,r))         i:PS(f,g,r)     prod:Y(g,r)
        v:V_FM(f,g,r)$(mf(f)*rm(r)*vom(g,r)*vfm(f,g,r))         i:PF(f,r)       prod:Y(g,r)
        v:V_XMD(i,s,r)$(vxmd(i,s,r)*rm(s)*(not vxm(i,s)))       i:P(i,s)        prod:M(i,r)
        v:V_VDA(i,r)$(RM(R)*vcm(i,r)*vdfm(i,"c",r))             i:P(i,r)        PROD:A(i,r)
        v:V_VIA(i,r)$(RM(R)*vcm(i,r)*vifm(i,"c",r))             i:PM(i,r)       PROD:A(i,r)

$offtext
$sysinclude mpsgeset gtap9

alias (i,i_,j_), (f,f_);
```

```
$macro  pnum    (P("c",rnum))
$macro DFM(f,g,r)       (V_FM.L(f,g,r)$(rm(r)*vom(g,r)*vfm(f,g,r)))
$macro DDFM(i,g,r)      (V_DFM.L(i,g,r) + V_VDA.L(i,r)$(vcm(i,r) and sameas(g,"c")))
$macro DIFM(i,g,r)      (V_IFM.L(i,g,r) + V_VIA.L(i,r)$(vcm(i,r) and sameas(g,"c")))
$macro REV_TO(g,r)      ((rto(g,r)*((P.L(g,r)*V_VDM.L(g,r))$(vom(g,r)-vxm(g,r))+\
                        (PE.L(g,r)*v_vxm.L(g,r))$vxm(g,r)))$(rm(r)*vom(g,r)))
$macro REV_TFD(i,g,r)   (rtfd(i,g,r) * P.L(i,r)  * DDFM(i,g,r))
$macro REV_TFI(i,g,r)   (rtfi(i,g,r) * PM.L(i,r) * DIFM(i,g,r))
$macro REV_TF(f,g,r)    ((rtf(f,g,r)  * v_fm.L(f,g,r) * ((PS.L(f,g,r))$sf(f)+\
                        (PF.L(f,r))$mf(f)))$(rm(r)*vom(g,r)*vfm(f,g,r)))
$macro XMD(i,s,r)       ((V_XMD.L(i,s,r)/vxmd(i,s,r))$(rm(r)*vim(i,r)*vxmd(i,s,r)) \
                        + X.L(i,s)$(rm(s) and rx(r) and vxmd(i,s,r)))
$macro PX(i,r)          ((P.L(i,r)$(not vxm(i,r)) + PE.L(i,r)$vxm(i,r))$rm(r) + PE.L(i,r)$rx(r))
$macro REV_TXS(i,s,r)   ((XMD(i,s,r)*rtxs(i,s,r)*vxmd(i,s,r) * PX(i,s))$(rm(s)*vxmd(i,s,r)*rtxs(i,s,r)))
$macro REV_TMS(i,s,r)   ((XMD(i,s,r)*rtms(i,s,r)*(PX(i,s)*(1-rtxs(i,s,r))*vxmd(i,s,r)  + \
                         sum(j_$vtwr(j_,i,s,r), PT.L(j_)*vtwr(j_,i,s,r))))$(rm(r)*vxmd(i,s,r)*rtms(i,s,r)))

gtap9.workspace = 1024;
gtap9.iterlim = 0;
$include gtap9.gen
solve gtap9 using mcp;

parameter       maxdev /1e-3/;
abort$(gtap9.objval > maxdev) "GTAP9 replication fails.", gtap9.objval;
```

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