Adventures in the Paraconsistent Jungle

Jean-Yves Béziau

Institute of Logic
University of Neuchâtel
Neuchâtel, Switzerland
jean-yves.beziau@unine.ch

Abstract

This is a survey of the main contributions of the author to the field of paraconsistent logic. After a brief introduction explaining how the author entered this field, Section 2 describes his work on C-systems: reformulation of the semantics of C1, creation of a sequent systems for C1, proof of cut-elimintaion for this system, extension of C1 into a stronger logic C1+. Section 3 is about his investigations on the definition of paraconsistent logic and a general theory of negation. Section 4 relates what he considers as its main contribution: the discovering that classical first-order logic and the modal logic S5 are paraconsistent logics and how this led him to a new theory of opposition, where a polyhedron replaces the traditional square of opposition. In Section 5 he explains that he conceives the philosophical aspects of paraconsistency mainly in relation with applications and he says a word of what kind of situation paraconsitent logic may apply to. Section 6 describes his work as paraconsistent promoter, editor, translator, and organizer. Finally in Section 7, he indicates his future lines of paraconsistent investigations and frothcoming works. The paper includes a complete bibliography of his works in paraconsitent logic. This paper can be read by anyone interested in logic even with few or no knowledge of paraconsistent logic.

Contents

1 Introduction

Faire de la vie un rêve et d'un rêve une réalité.
Pierre Curie

In this paper, [1] I tell my adventures in the Paraconsistent Jungle, from my discovery of paraconsistent logic in the Spring 1989 through the meeting with a donkey, to the encounter of strange creatures in Brazil, to the frequentation of the mysterious O-corner of the square of oppositon in California, to contructions of polyhedra on the bank of the lake of Neuchâtel.

As I already recalled in another paper [17], these adventures started in the Spring 1989 in France when I read a paper about Newton da Costa and paraconsistent logic in the French Lacanian psychoanalysis journal *L'Âne*. I was not a Lacanian and/or mentally sick, but I like to read any kind of magazines: from *The Notices of the American Mathematical Society* to *Les Cahiers du Cinéma*, from *National Geographic* to *Penthouse*, from *Cosmopolitan* to *The Economist*. I never had the ambition to be a one dimensional dull man. I like to watch was is going around and basically that's how I discovered Paraconsistent Logic.

In reading the donkey paper I was fascinated by this logic violating the principle of non contradiction, presented as a possible logic of the incouscious, as much as by da Costa's personality. I remember in particular that da Costa was quoting the above words of Pierre Curie and also there was a photo of him with a subtitle saying "I am not considering myself as a simple technician of logic."

At the beginning of 1991 da Costa was in Paris to bungee jump from the Ei el Tower. I had the opportunity to meet him and to show him works I have been doing on his systems (I didn't spend all my time riding donkeys). He liked it and he invited me to come to Brazil to work with him. Few months later I was in the Urban Jungle of São Paulo.

2 C-systems

Relação contraditória, portanto relação lógica,
Cadela que me prende, para me libertar ...
Lars Eriksen

2.1 Reformulation of the semantics of C1

I started to work on paraconsistent logic in Paris by studying the C-systems of da Costa, one of the first and most known systems of paraconsistent logic. My aim was to understand properly how C1 and its semantics work. This tentative led me to a reformulation of the axiomatization and semantics of C1. I tried to reformulate C1 in the most intuitive way.

[1] Work supported by a grant of the Swiss National Science Foundation

C1 was originally conceived by da Costa as an Hilbert-type proof system. In the orginal version, the only rule is *modus ponens*, and the axioms for C1 are the axioms of positive intiutionistic logic plus the following ones:

[10] $a \vee \neg a$
[11] $\neg\neg a \rightarrow a$
[12] $(b^o \rightarrow ((a \rightarrow b) \rightarrow ((a \rightarrow \neg b)) \rightarrow \neg a))$
[13-15] $a^o \wedge b^o \rightarrow (a©)b)^o$, $(© \in \{\vee, \wedge, \rightarrow\})$
[16] $a^o \rightarrow (\neg a)^o$

a^o means $\neg(a \wedge \neg a)$ A sound and complete semantics bivalent for this systen

was given by da Costa and Alves in 1976. This semantics is non truth-functional and the bivaluations are defined directly on the whole set of formulas. The conditions for conjunction, disjunction and implication are the same as the classical ones. Then we have the following set of conditions for negation.

⟦1⟧ if $\beta(a) = 0$, then $\beta(\neg a) = 1$
⟦2⟧ if $\beta(\neg\neg a) = 1$, then $\beta(a) = 1$
⟦3⟧ if $\beta(b^o) = \beta(a \rightarrow b) = (a \rightarrow \neg b) = 1$, then $\beta(\neg a) = 0$
⟦7⟧ if $\beta(a^o) = \beta(b^o) = 1$, then $\beta(\neg(a©b)) = 1$, $(© \in \{\vee, \wedge, \rightarrow\})$

The first of these conditions is half of the standard one for classical negation, and generates the excluded middle given by axiom [10]. The meaning of the other conditions is not necessarily clear. They look like *ad hoc* conditions to get completeness. Obviously, condition ⟦2⟧ is a semantical description of axiom [11], condition ⟦3⟧ of axioms [13-15] and condition ⟦7⟧ of [16]. I succeed to show that we can replace these artificial conditions by a set of intuitive conditions $[\![C1 - JYB]\!]$, which are the following:

$[\![\neg_{lo}]\!]$ if $\beta(a \wedge \neg a) = 1$, then $\beta(\neg(a \wedge \neg a)) = 0$
$[\![\neg_{l©}]\!]$ if $\beta(a©b) = 1$ and $\beta(a) = 0$ or $\beta(\neg a) = 0$ and $\beta(b) = 0$ or $\beta(\neg b) = 0$, then $\beta(\neg(a©b)) = 0$, $(© \in \{\vee, \wedge, \rightarrow\})$
$[\![\neg_{l\neg}]\!]$ if $\beta(\neg a) = 1$ and $\beta(a) = 0$ or $\beta(\neg a) = 0$ then $\beta(\neg\neg a) = 0$

These conditions are all restricted forms of the other half of the condition which defines classical negation, i.e. if $\beta(a) = 1$, then $\beta(\neg a) = 0$. This condition is a semantical formulation of the principle of non contradiction, it says that a formula and its negation cannot both be true. The principle of non contradiction does not hold in general in C1, but the idea of C1 is not just to withdraw half of the semantical condition which defines classical negation, which leads to a quite weak logic, but to replace it with a restricted version expressed by the conditions $[\![C1 - JYB]\!]$.

The condition $[\![\neg_{l©}]\!]$ says that if the principle of non contradiction holds for two formulas a and b it also holds for the conjunction, disjunction and implication. The condition $[\![\neg_{l\neg}]\!]$ says that if the principle of non contradiction holds for a formula a it also holds for its negation. These two conditions can be unified by some notational device and can be expressed inutuitively as meaning: the principle of non contradiction is preserved by complexification. This is one of the two main ideas of the semantics of C1.

The other main idea is given by condition $[\![\neg_{lo}]\!]$ which can be described as: a contradiction obeys the principle of non contradiction. This condition allows to have a classical negation within C1 and therefore to translate classical logic into C1.

In my reconstruction, I introduced Ci which is the logic semantically defined with classical conditions for conjunction, disjunction, implication plus $[\![1]\!]$, plus $[\![\neg_{lo}]\!]$. Ci is a very weak logic, it only allows to define a classical negation with the paraconsistent negation. If we add the preservation principle, we get much more: double negation and some de Morgan laws.

Having reformulated the semantics of C1, I was also able to give another more intuitive Hilbert system for C1. In this system we have *modus ponens*, plus the axioms for *classical* positive logic, plus the following ones:

$a \vee \neg a$
$(a \wedge \neg a) \wedge \neg(a \wedge \neg a) \rightarrow b)$
$a^o \wedge b^0 \rightarrow (a \copyright b)^0$
$a^o \rightarrow (\neg a)^0$

For Ci, we just have to take the axioms for classical positive logic plus the first two axioms above.

With this new formulation of C1 I was able on the one hand to describe in a more intuitive way the method of truth-table for C1 and to give some set-theoretical diagrammatic representations of how it works , on the other hand to built a sequent systems for C1. I will not talk about the first point here. The second point is the subject of the next section.

2.2 Sequent systems for C1 and cut-elimination

Andrès Raggio, an Argentinian logician former student of Paul Bernays, had tried to formulate a sequent calculus for C1 in the 1960s, but he didn't quite succeed. The formulation I gave in 1990 is based on my reformulation of the semantics of C1 presented in the preceding section and on the discovery of an intimate connection between sequents and bivaluations described in details in My PhD and in .

The sequent system S1 for C1 has the same rules as the sequent system for classical propositional logic, except the rules for negations which are the following:

RULES FOR NEGATION IN $\mathcal{S}_1$

$$[\neg_r]\ \frac{a \Rightarrow}{\Rightarrow \neg a} \qquad \frac{\Rightarrow a \wedge \neg a}{\neg(a \wedge \neg a) \Rightarrow}\ [\neg_{lo}]$$

$$\frac{\Rightarrow a \copyright b \qquad a, \neg a \Rightarrow \qquad b, \neg b \Rightarrow}{\neg(a \copyright b) \Rightarrow}\ [\neg_l \copyright]$$

$$\frac{\Rightarrow \neg a \qquad a, \neg a \Rightarrow}{\neg\neg a \Rightarrow}\ [\neg_{l\neg}]$$

In S1 we have all the structural rules, in particular the cut rule. But this rule can be elimated. Cut-elimination of S1 is not a big deal. You just have to check that these monstruous rules do not interfer with the standard process. By studying cut-elimination for S1 I was able to give a general formulation of the cut-elimination theorem in my PhD.

S1 rules don't have the subformula property, but they have something analogeous: the subnegformula. A subnegformula of a formula is a subformula or a negation of a proper subformula. From this property and cut-elimination, we have another method of decidability for C1 using S1.

All this work about C1 appears in my Master's thesis [?] and was subsequently published in [?].

2.3 C1+

There is an obvious and intuitive way to strengthen C1 in a logic C1+. I had this idea in 1990 and it appears in several of my works, in particluar [?]. The idea is to replace the axiom

$a^o \wedge b^0 \rightarrow (a©b)^0$

by the axiom

$a^o \vee b^0 \rightarrow (a©b)^0$

This corresponds to a new preservation principle: in order for the principle of non contradiction to be preserved by complexification, we just need that one of the proper subformulas of a complex formula obeys the principle of non contradiction.

With this strengthening we get 4 more de Morgan laws. In my paper I presented the following comparative table of the logics C1, C1+ and their paraconsistent duals. In this paper I used the notation Cx, C+ instead of C1 and C1+, to emphasize the conjonctive (x) or disjunctive (+) formulation of the preservation principle.

Cx x	+ C+
$\neg(a \wedge b) \vdash \neg a \vee \neg b$	$\neg(a \vee b) \vdash \neg a \wedge \neg b$
$\neg(\neg a \wedge \neg b) \vdash a \vee b$	$\neg(\neg a \vee \neg b) \vdash a \wedge b$
$\neg(a \wedge \neg b) \vdash \neg a \vee b$	$\neg(a \vee \neg b) \vdash \neg a \wedge b$
$\neg(\neg a \wedge b) \vdash a \vee \neg b$	$\neg(a \vee b) \vdash a \wedge \neg b$
C	C
K	K
$\neg a \wedge \neg b \vdash \neg(a \vee b)$	$\neg a \vee \neg b \vdash \neg(a \wedge b)$
$a \wedge b \vdash \neg(\neg a \vee \neg b)$	$a \vee b \vdash \neg(\neg a \wedge \neg b)$
$\neg a \wedge b \vdash \neg(a \vee \neg b)$	$\neg a \vee b \vdash \neg(a \wedge \neg b)$
$a \wedge \neg b \vdash \neg(\neg a \vee b)$	$a \vee \neg b \vdash \neg(\neg a \wedge b)$
Kx x	+ K+

The table has to be read as follows: what hold for the x-systems, hold for the +-systemes (but not the contrary). What hold for the K-systems do not hold for the C-systems and vice-versa.

In [?] I also introduced the notion of non truth-functional many-valuedness. I present non truth-functional semantics for all these systems. This allows to get back the subformula property.

3 Negation theory

There are no paraconsistent negations.
Barry Slater

One thing that really interests me in paraconsistent logic is that it gives a better idea of what negation is, and it gives in particular a better idea of what classical negation itself is and is not. From the point of view of someone who is trying to understand negation, there is no opposition between classical logic and paraconsisten logic. My work in paraconsistent logic naturally led me to a general study of negation.

3.1 Definition of paraconsitent negation

This started with the question of definition of paraconsistent logic. I was interested in this problem since the beginning and in particular I had read with interest the excellent paper of Igor Urbas, where he gave an improved version of the standard definition of paraconsistent negation. The standard definition is based on the rejection of the *ex-contradictione sequitur quodlibet* (EC hereafter): $a, \neg a \vdash b$. But Urbas remarked that we want to reject more quodlibet than the usual quodlibet, for example we want also to reject: $a, \neg a \vdash \neg b$.

The definition of Urbas is certainly a great improvement of the standard definition, but it is still not a very satisfactory definition, since what we want is not only a (reinforced) negative criterium for paraconsistent negation, in order to have a *paraconsistent* negation, but we want also a positive criterium, in order to have a paraconsistent *negation*. Many unary operators not obeying Urbas' version of EC, have absolutely nothing to do with negation, for example the identity operator which transforms any formula into itself. From this viewpoint, any logic is paraconsistent(since a paraconsistent logic is defined as a logic in which there is at least one paraconsistent negation).

I think there is a great deal of confusion, hypocrisy and crookedeness in defining paraconsistent logic only negatively and talking as there were no problems with paraconsistent negation. I discussed this question mainly in two papers, one presented at the First World Congress on Paraconsisteny in Ghent in 1997, entitled "What is paraconsistent logic?" and the other presented at the Second World Congress of Paraconsistency in Juquehy in 2000, entitled "Are paraconsistent negations negations?". Both are related to a paper written by Barry Slater, "Paraconsistent logics?" published in *the Journal of Philosophical Logic* in which he doubts that there are any paraconsistent negations. Although Slater is wrong, as I explained in a paper which is a reply to his, "Paraconsistent logics!", he found the Achilles' heel of paraconsistentists and rightly tickled it.

In my Juquehy's paper I argue that the existence of paraconsistent logic has not been proved so far (a position which is neither Slaterian nor anti-Slaterian). I also developed a classification of properties for negation and of the so-called paraconsistent logics.

3.2 Pure negation and incompatibility results

Classical negation can be formulated with only one axiom, the *reductio ad absurdum*, but this axiom can be decomposed into a multiplicity of weaker axioms. In my paper "Théorie législative de la négation pure" [?], I made a study of negation alone, independently of other connectives (implication, conjunction, etc.). I showed that with very few assumptions on the consequence relation, basic properties such as *contrapositio*, *weak reductio*, etc. have to be rejected if one rejects EC. This study gave me a rather pessimistic vision of paraconsistent logic. If we want a paraconsistent negation defined by Urbas' version of EC, then nearly no properties of negation are compatible with this rejection. Positive properties have to be found within the interaction with other connectives, in particular with conjunction and disjunction, such as de Morgan's laws.

In a paper published in *Notre Dame Journal of Formal Logic* [?], I showed a basic incompatibility result which applies both to Asenjo's logic A3, Priest's logic LP and the logic J3 of da Costa and D'Ottaviano. In a paraconsistent logic, If $\neg(a \wedge \neg a)$ is valid and also the double negation is valid, then the replacement theorem cannot hold. In fact the replacement does not hold in A3, LP and J3, it seems I was the first to realize that, a typical counter-example is that $\neg(p \wedge \neg p) \dashv\vdash \neg(q \wedge \neg q)$ but not $\neg\neg(p \wedge \neg p) \dashv\vdash \neg\neg(q \wedge \neg q)$. So from the point of view A3, LP and J3 are no better systems than the C-Systems (the failure of the replacement theorem in C1 and related systems is traditionally considered as fatal defect, see for an analysis of this problem).

4 Modal logic, paraconsistent logic and the square of oppositions

Ça crevait les yeux,
mais la poufiasse n'y avait vu que du feu.
Arthur Oursipan, *Panique à Rio*

4.1 Classical logic is paraconsistent

I can say that my main contribution to paraconsistent logic, up to now, is the discovering that classical logic is paraconsistent. I know that this sounds terribly paradoxical, and that even the most crazy paraconsistent logicians, those living in the antipodes, are not ready to consider this as true. But it is. In the next subsections I tell the story of my discovery that the modal logic S5 and classical first logic are paraconsistent logics, and di erent consequences of this discovery.

My impression is that this discovery was not very welcomed and since the time I discovered this, about five years ago, few people have talked about it, although I have presented lectures on this topic in many places around the world: Moscow, Amsterdam, Melbourne, Rio de Janeiro, Naples, Vancouver, etc. I think the reason is that such fact is disgusting both for paraconsistentists and anti-paraconsistentists.

On the one hand, people believing in contradictions or wanting to built logics in which from a contradiction not everything is derivable, have been working hard during many years to built new logics, sometimes quite complicated, not realizing that there were very simple logics already at hand able to do that, namely good old classical first-order logic and the most well-known modal propositional logic, S5.

On the other hand, for a classicist, like the late Quine, believing that the rejection of the law of non-contradiction is a "popular extravaganza" (*Philosophy of logic*, p.81), realizing that there is a paraconsistent negation within classical logic is a shocking obscene reality, similar e.g. as realizing that his wife is a hermaphrodite. Quine wrote "the classical logic of truth-functions and quantification is free of paradox, and incidentally it is a paradigm of clarity" (*Ibidem*, p.85). He could never have imagined that the beautiful classical fruit was infected by a logic of paradoxes.

4.2 The origin of the discovery of this amazing fact: Fortaleza, Copacabana and Torun

I discovered this fact several years ago. It is connected with things I had been thinking of since more than fifteen years. But things took shape in 1997 in Fortaleza. At the Beira Mar, I was discussing with my friend Arthur de Vallauris about the notion of contradiction, he was repeating a sufi story with had inspired him in his research in paraconsistent logic. What I was saying to him, is that a thing may appear black to someone, and white to someone else, that doesn't mean that it is black and white. In the "logic of appearances" (expression coined by our friend Tarcisio Pequeno), an object may have contradictory properties, which correspond to di erent viewpoints. I derived such conceptions from a study I had made on the work of the physicist David Bohm at the end of the eighties. In his book *Wholeness and the implicate order*, he gave several metaphors supporting such a view.

So my idea was to develop a logic that can be used to reason with di erent contradictory viewpoints. The idea was that the negation of a proposition is false only if the proposition is true from all viewpoints. If from all evidences it is true that light is a wave, then it is false that light is not a wave. On the hand, if according to some experiments it is true that light is a wave and according to some other experiments, it is false that light is a wave, then what we can say from the point of view of all experiments is that it is true that light is a wave and that it is not a wave. What light really is, only God knows. After my discussion with Arthur, when I was back to my room at the Hôtel Olympo, I started to develop a possible worlds semantics based on this idea.

When I was back to Rio, were I was living at this time, I developed a full study of the related logic, that I called Z. I saw a connection between S5 and Z. The paraconsistent negation of Z was equivalent to $\neg\Box$. From this I infered that it was possible to define Z into S5, that Z was in S5, that therefore S5 was paraconsistent and that also, via Wasjberg's theorem, classical first-order logic was paraconsistent. I discussed my discovery with the Italian logician Claudio Pizzi, who has a little castle in Copacabana and uses to come there frequently. Pizzi told me two important things: one right and one wrong. The wrong was that the operator $\neg\Box$ corresponds to contingency, the right was that S5 and Z are equivalent since it is possible in S5 to define classical negation and necessity with $\neg\Box$, conjunction and implication.

The intuition underlying Z is connected with the basic intuition of Jaśkowski's discussive logic. Jaśkowski wanted to formalize the logic of a discussion group where people may have contradictory opinions. To formalize this he switched the universal-like definition of truth in a Kripke Structure by a existential-like one: a proposition is true in a Kripke Structure i it is true in some worlds. This leads to a logic which has some quite strange properties (non adjunction). It seemed to me that my formalization was a fomalization of Jaśkowski's intuition, which led to a more interesting paraconsistent logic that the standard formulation of Jaśkowski's logic. I emphasized this fact in a lecture I gave on the logic Z in July 1998 in Torun, Poland, at the conference commemorating the 50 years of Jaśkowski's paper.

4.3 The mysterious 0-corner of the square of oppositions

I made further important discoveries about the paraconsistent negation $\neg\Box$ when I was in California (I had moved to Stanford at the beginning of 2000). In 2001 I was attending Johan van Benthem's seminar on modal logic and he draw the square of modalities, with the four corners $\Box$, $\Diamond$, $\neg\Box$ and $\neg\Diamond$ saying that there was no name for the modality $\neg\Box$, and that was the same for the square of quantifiers, there was no name for $\neg\forall$. Few days later I attended a lunch lecture at the CSLI by another Dutch, Pieter Seuren, about quantifiers, and he was talking about the non-lexicalization of this O-corner of the square: linguists had been studying many languages around the world and never found a primitive word for the quantifier $\neg\forall$. The conclusion of Seuren was that the square of opposition was a wrong theory of quantification.

It was very funny to see that paraconsistent negation was just showing its nose at this mysterious O-corner. I decided to explore the paraconsistent negation within this square problematic and I made quick advances, due to the fact that I was able to put several di erent information together and complete the jigsaw. I had read some years ago something interesting on the square of opposition by Robert Blanché and I decided to look back at Blanché. The analysis of the square of opposition by Blanché is something very nice and profound. He suggests to replace the square of the quantifiers by a hexagon made of two triangles, one of contrariety, one of subcontrariety. The six vertices of the hexagon represents six quantifiers. If we made a similar construction for modalities, we have an hexagon with six modalities, and we see clearly where stands contingency, it is the vertice $\Diamond \wedge \neg\Diamond$ of the triangle of subcontrariety. This modality and its negation (non contingency) have been studied by Montgomery, Routley at the end of the 1960s and more recently by Humbertsone and Kuhn. It is clearly di erent from $\neg\Box$. $\neg\Box$ is the dual of $\neg\Diamond$, the impossible, which corresponds to intuitionistic negation in S4, and which is in general a paracomplete negation.

So in the hexagon of modalities appear clearly two negations, but what do not appear are the relations between these negations and a proposition *tout court*. My idea was then to introduce the proposition *tout court* in the hexagon, and by symmetry, the classical negation of the proposition. Then I transformed the hexagon of Blanché into an octagon. In this octagon, the relation between a proposition and its paraconsistent negation $\neg\Box$ is a relation of subcontrariety and the relation between a proposition and its paracomplete negation $\neg\Diamond$ is a relation of contrariety. All this shows very clearly that the three notions of oppositions met in the square of Aristotle correspond to three negations: contradictory to classical negation, contrariety to paracomplete negation, subcontrariety to paraconsistent negation. My impression was that square of opposition could be used as a philosophical basis for a theory of negation admitting various type of negations, including paraconsistent negation.

I made some further progress in this square approach to paraconsistent negation at the beginning of 2003 in Switzerland (where I moved in august 2002). I had never been very satisfied by the octagon because it was not so simple and nice as Blanché's hexagon. I was trying to put everything together in a more beautiful geometrical objet. I discovered then that the octagon can be decomposed in three hexagons having the same structure as Blanché's hexagon, i.e. being the composition of two triangles. Among these three hexagons, there is Blanché's hexagon itself, a paraconsistent hexagon in which appears the paraconsistent negation and a paracomplete hexagon in which appears the paracomplete negation. Then my idea was to put these three hexagons together by constructing a three dimensional object, a polyhedron. One possible polyhedron is the first stellation of the rombic dodecahedron. But my friend Alessio Moretti has contructed a more interesting polyhedron. In this polyhedron appears a fourth hexagon that he and Hans Smessaert discovered. This polyhedraic constructions are a great achievement which gave an exact idea of the place of the paraconsistent negation $\neg\Box$ and its relation with other negations and modalities. Moreover these relations can be understood easily through geometrical intuitons, you just have to look at the objets. Due to the ridiculous limitations of Latex, I will not present pictures here, but they can be found on my website at www.unine.ch/unilog.

5 Philosophy and applications of paraconsistent logic

Endive aumenta o colesterol. Ovo, manteiga, bacon fazem baixar o colesterol,

é a última descoberta dos pesquisadores de uma universidade sueca.
Rubem Fonseca *Idiotas que falam outra lingua*

For me, philosophical aspects of paraconsistent logic are directly related with the question of its applications. I consider philosophy as something related with reality not with Nephelokokkugia (The city of cuckoos in the clouds). As it is known, it is not necessary the case of philosophers in general and not either the case of logical philosophers. Paraconsistent logicians are most of the time not interested in a world where there is an Ei el Tower in Paris, black birds in the sky of Hiroshima, nice girls on Copacabana beach. They prefer to live in a world full of "funny" paradoxes or, instead of looking at the world at it is, they try to reduce it to a terrible machine ruled by the God of Contradiction, where there are contradictions everywhere: between night and day, cats and dogs, cheese and wine. Many people are interested in paraconsistent logic because they think that contradiction is the "thing". They don't want to eliminate contradictions, they want to play with them. And if you say that there are no contradictions, if you take out their toys, their daily bred, they become hungry like spoiled children.

In di erent papers I wrote, I tried to provide interpretation of paraconsistent logic, by giving some examples. It is striking to see that most of the time, formal logicians give only very few examples or trivial ones (Snow is white, Bush is a pacifist, Bachelors love married women). Unfortunately paraconsistent logicians are no exceptions. So we have the impression that what they are doing is some silly formal games, without meaning, with no relations with everyday life. In the best case, when they are related to something, it is to some "funny" paradoxes that nobody care about except those leaving in Nephelokokkugia.

It is a pity because one of the strongest features of paraconsistent logic is its potential application, not to metaphysics, but to technology. Inconsistent databases are a "reality". We need paraconsistent logic to construct robots able to behave in a surrounding inconsistent world of information. Such robots in fact have already been conceived, the first of them is a female called Sophia, born in Brazil in 1999. She had now a little sister, Emmy.

In the di erent examples I have been developed, I was especially inspired by Flaubert. In his last unfinished book *Bouvard et Pécuchet*, Gustave Flaubert describes the adventures of two sympathetic old Parisians who decided to retire in the countryside and study all sort of things: physics, agriculture, how to cure a hysterical woman, etc. In each case, they try to get some information, either by reading books or asking some people, and in each case they always find contradictory information, so at the end they don't know what to to, what to think. This is expressed by Flaubert in a very funny style, characterized by Maupassant as "juxtaposition antithétique".

In [24] I gave some examples of contradictory information given by two fictitious Physicians, Dr. Bouvard and Dr. Pécuchet. Medecine is a rich source of examples for paraconsistent logic, we find contradictions everywhere, contradictions between diagnoses, contradictions between theories (homeopathy vs. allopathy), etc. Medecine was a also favourite target of Flaubert's irony (cf.

the couple physician-pharmacian Bovary-Homais in *Madame Bovary*.)

Imagine that Rintintin is sick. On the one hand Dr.Bouvard tells him he has a genuine disease, like cancer, and that he will irremediably die before Easter, on the other hand Dr. Pécuchet tells him it is a benign disease, that he has just to put his feet on hot water everyday while smoking a special herbal mix. So what will Rintintin do? Put his feet on hot whater while smoking everyday this special herbal mix until Easter and see if he will die? At this point, it may be useful for him to use paraconsistent logic...

Other applications I work with are related with justice and law. Imagine that someone has been killed and that Pécuchard has been seen close to the place of the crime at the time it happened. Imagine also that another witness has seen Pécuchard at this same time in the zoo giving bananas to the monkeys. Here again paraconsistent logic can be useful and can avoid to believe a witness rather another one on the basis of subjective criteria, like for example the guy who has seen Pécuchard at the zoo is a "crazy" guy doing research on iterative forcing.

6 My contribution to the development of paraconsistent logic

When I started my researches in paraconsistent logic at the end of the eighties, it was an exotic topic known only by the happy few. It is amazing how the situation has changed 15 years.

I have myself contributed to the development of paraconsistent logic, not only by writing papers, but also by discussing with people, organizing events and doing some editorial work.

In France, when I started to work on paraconsistent logic, very few people knew what it was. Most of the people never had heard this curious name. This was the case of Daniel Andler, who nevertheless thought that the subject was interesting and accepted to directed my Master on the subject. By studying da Costa's papers, I discovered that in the early 1960s he had collaborated with a French mathematician from Clermont-Ferrand called Marcel Guillaume. I asked Andler if he knew something about Guillaume, if he was still alive, etc. He knew him and told me that he was still in Clermont-Ferrand. I wrote to Guillaume and he was kind enough to answer me and sent me some papers he wrote with da Costa I was not able to find in Paris. He also told me that he was not working in the subject since many years.

When I met da Costa in Paris in 1991, he was also not really working anymore on paraconsistent logic since several years. His main interest was on the applications of logic to physics, in particular versions of Gödel's theorem for physics.[2] And when I went to work with him in São Paulo, there were very few

[2]He got some positive results on this direction together with the Brazilian physicist, F.A.Doria, who never wrote papers on paraconsistent logic, but is one of the most paraconsistent being in the world.

people working with him on paraconsistent logic. But slowly people started to be interested again in paraconsistent logic, in particular da Costa himself. In some sense I contributed to the revival of paraconsistent logic in Brazil and also to its spreading all over the world: after spending one year in Brazil I did some researches in Poland and California and gave lectures on paraconsistent logic in many countries. I gave also several mini-courses on paraconsistent logic to advanced students: at the ENS in Lyon in 2000, at the ESSLLI in Birmingham in 2000, at the XXI SBC in Fortaleza in 2001, etc.

I translated into French, or something alike, the book of da Costa *Ensaio sobre os fundamentos da logic*. The book in French is called *Logiques classiques et non classiques* and was published by Masson in 1997. This book is not only about paraconsistent logic, but as you guess, paraconsistent logic is widely presents in it. There is in particular a discussion about Lukasiewicz's analysis of Aristotle's analysis the principle of contraciction. There is also a discussion about paradoxes and what da Costa calls Hegel's thesis, the thesis according to which there are true contradictions. In this book da Costa writes: "On the one hand, there are some people for whom contradictions play a quasi mystical rôle, being used as explaining nearly all the universe; on the other hand, there are some excellent specialists who believe that contradiction is something ununderstable. Paraconsistent logic not only contributes to demystify contradiction but also to calm down those who are afraid of it."

The French edition is a revised version of the original Portuguese edition with furthermore two appendices I wrote myself. One of them is on paraconsistent logic. I tried to present the logic C1 in a intuitive way, with a natural deduction system and providing some examples.

I edited two special numbers of *Logique et Analyse* dedicated to Contemporary Brazilian Research in Logic containing several papers on paraconsistent logic. The first volume contains a paper of Guillaume recalling his adventures in snowy Curitiba at the beginning of the 1960s. The second volume is quite unique because you can find in it a paper by Quine standing by some papers on paraconsistent logic. Quine's paper itself is not, as you guess, on paraconsistent logic, but it is called "Mission to Brazil" and is related with the year Quine spent in Brazil in 1942-43 and the book which was the fruit of this stay, *O sentido da nova logica*, a book Quine wrote in Portuguese or something alike, and which has never been translated in English.

I participated in a significant way to three paraconsistent events. The first of them is the WCP2 (Second World Congress on Paraconsistency) which was organized in Brazil in 2000, for the commemoration of da Costa 70th birthday, following the WCP1 (the 1st World Congress on Paraconsisteny) organized in Ghent in 1991. I was part of the scientific committee of the WCP2, but my main contribution was the finding of a very nice location for the realization of the congress. I did that with my friend João Marcos, we spent about two days driving on the littoral of the State of São Paulo to find a beautiful place and we choosed Juquehy Beach. It seems that everybody liked the place. The cousin of da Costa, Lars Eriksen, told me it was the best congress of his life and that he particularly enjoyed the food.

Together with Walter Carnielli, I was the main organizer of the WCP3 which happened in Toulouse, France, 2003. I organized also a small paraconsistent event in Las Vegas, USA in 2001.

7 My future work in paraconsistent logic

Eu que nao me sento no trono de um apartamento,
com a boca escancarada, cheia de dentes, esperando a morte chegar.
Raul Seixas *Ouro de tolo*

I think I will work on paraconsistent logic until the end of my life, and maybe after, if there are any possibilities to do logical researches in paradise or/and hell. Here are my projects for the next few years.

I am presently writing a book on paraconsistent logic, with my PhD student Alexandre Costa Leite. The title of the book is *A Panoramic introduction to paraconsistent logic*.

At the present time, there is no introductory book of paraconsistent logic, where someone can learn the basic techniques of paraconsistent logic. I think this is a very bad thing which reflects the confuse situation prevailing at the present time in paraconsistent logic: paraconsistentists talk as if paraconsistency was obvious, as if there were a lot of satisfactory and well-known systems of paraconsistent logic. But it is not at all the case. A typical example is that, as pointed by João Marcos, most of time when someone is talking about Jaśkowski he doesn't know what he is speaking about. There is no clear definition of what is Jaśkowski's logic. So if someone wants really to learn what is paraconsistent logic, how it works and not just to read a lot of pseudo-philosophical blah blah blah, or some very technical papers, he has nothing at hand.

Paraconsistent logic is not only one logic as is intuitionistic logic, neither a multiplicity of logics constructed with one technique like modal logics. This is why paraconsistent logic is not something easy to study. Our book will reflect this state of a airs. In *A Panoramic introduction to paraconsistent logic*, each chapter will present a di erent paraconsistent logic based on a di erent technique. Through the book many examples will be given. And in a final chapter we will present a general theory of paraconsistent negation.

A line of investigation I will develop is related to the square of opposition and paraconsistent negation as $\neg\Box$. My work can be developed in several directions, that can be explored with the geometrical intuitions given by the polyhedra. One direction is the study of $\neg\Box$ and the related polyhedra in the cases of some specific modal logics. Alessio Moretti has been studying the case of S4. In S4, and in each modal logic, $\neg\Box$ behaves as a paraconsistent negation. So there are a lot of paraconsistent negation to be (re)discovered. I am also studying the behaviour of $\neg\Box$ in modal logics based on many-valued semantics.

Another direction is the generalization of the Star-of-David that can be found in Blanché's hexagon into a star with eight vertices constructed by the combination of two squares (This symbol is quite popular in the Muslim world, I saw it for example in the great mosque Hagia Sophia in Istambul). One square is a contrary square, and the other one is a subcontrary square, they generalize the contrary triangle and subcontrary triangle of Blanché's hexagon. In the same way that Blanché's Star-of-David is based on trichotomy, the Muslim-Star is based on quatritomy and we can go on and constructed stars based on any politomy. This allows to develop a general theory of polytomy and shows that Kant was totally wrong when saying that only dichotomy is non empirical and that polytomy cannot be teached in logic (*Logik, 113.*)

I have also many other di erent projects, working on some specific paraconsistent logics, like De Morgan logic, writing on the philosophical aspects of paraconsistency, developing connexion between paraconsistent logic and universal logic, etc.

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