Using and Learning Vision-based Self-positioning
for Autonomous Robot Navigation

Claudio Facchinetti and Heinz Hügli
(`claudio.facchinetti@imt.unine.ch`)
Institute of Microtechnology
University of Neuchâtel, Switzerland

Abstract

We propose in this paper a new class of vision-based behaviors that provide navigation with self-positioning to an autonomous mobile robot. Self-positioning is performed by servoed homing behaviors that use simple visual features of the environment as homing sites, such as landmarks, wall corners or ceiling structures. We show that self-positioning provides means for the robot to learn the spatial structure of unknown environments, by building a map of homing sites and paths between them. Unlike other localization methods, a geometric reconstruction of the environment from the sensors is not needed. Three implementations of homing behaviors as well as results from experimental tests using a real robot are presented.

1   Introduction

One of the problems autonomous mobile robots are confronting is knowing the robot position and orientation (or configuration) with respect to an internal description of the environment. It is often observed that using odometric sensors is not sufficient for controlling the robot: additional information on the environment is necessary. This can be obtained by adopting exteroceptive sensors that react to features of the environment, such as tactile sensors, range-finders and vision sensors.

Using stereoscopy (2 or 3 cameras) or dynamic vision (single mobile camera) techniques, a geometric model of the environment can be reconstructed and matched against a predefined model (a map) in order to estimate the robot position. This is known as the positioning approach, which is essentially robot-centric: it consists in reconstructing the environment from the robot sensors point of view. This is a rather complex problem that requires heavy computation and for which most of the efforts while trying to solve it generally aim at coping with the unstable nature of both robot resources and the real world.

Reconstructing a geometric model of the environment is not always needed. Simple tasks such as following a wall or going to an object may be amenable to behavior-based approaches. Behaviors may control the robot by servoing its moves to low-level visual primitives, such as points and segments extracted from image sequences, that are related to features and structures of the environment. However, a common problem of the behavior-based approach is that it is difficult to solve typical navigation problems such as "bring a chair to the cafeteria", because the robot interacts with the environment via reactive behaviors that are not mapped in space [3].

We propose a new class of vision-based behaviors that provide navigation with self-positioning to autonomous mobile robots using servoing techniques. We understand here self-positioning (or homing) as the action of finding a stable state relatively to the environment in terms of visual primitives in an image or a set of images. Servoing can be realized by means of visual- or position-based approaches.

Visual servoing is fully world-centric: the environment drives the robot via its sensors. Hence, no complex scene reconstruction is required and the robot moves are directly bound to the visual primitives in the sensed image(s). Position servoing needs to estimate the robot sensor position and orientation relatively to an environment feature, by interpreting the visual information of the object as captured in one or more images. Although no true geometric reconstruction of the environment is engaged, accurate models of the camera and the robot and strong features such as the object distance from the camera or the object dimension are required.

Self-positioning may have different aspects depending on the sensor that is used for the implementation. *Docking*, which is sometime used to describe the specific action of driving the robot to a power supply base, is a special case of self-positioning. We developed several servoed behaviors that home the robot relatively to reflective landmarks, ceiling structures or

wall corners (see Figure 1).

Figure 2: *Three basic reasoning layers that control the robot actions.*

Figure 1: *NOMAD robot self-positioning on a corner site using a horizontal laser line-stripping vision sensor (which trace is visible on the corner of the wall). A second camera is placed vertically for self-positioning on ceiling structures.*

The ability of learning the environment structure for an autonomous robot is critical, since manual input of features and structures most natural for humans may not correspond well to features and structures to which the robot has sensory access, especially within a behavioral context. *Learning* relates here to characterizing new homing sites, as well as building a map of sites in the environment and paths between them for navigation. An advantage of using self-positioning is the independency and modularity of the homing behaviors. All the details concerning a particular self-positioning implementation are handled internally and may be easily changed without affecting other elements in the behavioral architecture. Furthermore, the noisy and unstable nature of sensors does not have to be handled directly in the learning process.

Positioning, localizing and the self-positioning approach we propose here are represented in a rather basic hierarchical model in Figure 2, for which each layer needs the underlying structure to be functional. The robot control is split in three reasoning layers with increasing execution times: the low-level control layer binds the robot sensors and actuators in a loop that is assumed continuous; the mid-level stimulus/response automaton operates at fixed periods of time, while the top-level reasoning layer needs multiple periods to anticipate and plan present and future

actions. Self-positioning servoes the robot actuators to visual primitives and may be associated with the bottom control level. Positioning operates discontinuously due to heavy computational needs (mainly for scene reconstruction) and corresponds to the mid-level reasoning level, while localization has to plan possibly multiple strategies for recovering the robot position in an input map and corresponds to the top reasoning layer.

The rest of this paper is organized as follows. The following section retraces related work in vision-based mobile robotics. Section 3 presents the vision-based architecture we developed for our mobile autonomous robot. Section 4 details homing sites and their characterization. Section 5 briefly presents a simple scheme for navigating using learned homing sites. Examples of homing behaviors that we developed and tested on a real robot are presented in section 6. Finally, section 7 concludes this paper.

2   Related work

The problem of localizing and positioning the robot in a predefined input map has been widely studied. Localization (or self-localization) consists in recognizing the environment and is usually first applied for correctly initializing a positioning loop. Positioning often use sensing techniques based on stereoscopy [12] or dynamic vision [11, 20]. A drawback is that handling sequences of images (from multiple or mobile cameras) for reconstructing a model of the environment leads to the difficult problem of sensor fusion [6, 14]. Hence, probabilistic methods are usually applied for weighting new sensed data based on an estimate of their reliability [27]. Other approaches based on non-probabilistic methods propose simplified models, such

as the set membership principle, for handling the same data [5, 24]. As reported in [10], the weakness of these methods is that they do not capture the relevant physics of the robot resources. A sensor will most of the time yield good or even excellent measurements of environment features, but sometime return irrelevant data. This unstable behavior has been extremely difficult to capture in any robust model.

A study of robot homing using combinations of model views is presented in [7]. Two methods are discussed. The first one computes the position of the robot in an input map of the environment to determine the direction to a target. The second method is adaptive; it uses a predetermined homing pattern that is aligned with a model to compute the direction to the target (positioning is not required). More generally, an overview of related work in vision for mobile robots shows that most of the efforts have been brought to 3D or 2D geometric reconstruction, but that little work has been dedicated to visual or position servoing for autonomous mobile robots. It seems however particularly interesting to build simple behaviors based on vision that servo robot moves in order to get real-time interaction with the environment.

Among work in position and visual servoing for robot manipulators, various problems have been raised concerning the modelling of camera motion relatively to signal variations in the image [17, 18]. The importance of correctly trimming the models the servoing behavior interaction has also been pointed out [19, 23]: a slight variation in this matrix and the end effector position should result in a slight variation in the image. Position servoing implementation are often dynamic, with acquisition and interpretation steps run simultaneously [15, 16]. In [25], an adaptive technique is used for the control of a robot constrained to 3 degrees of freedom. Position servoing has been mainly applied to robot manipulators control with a camera mounted on the end effector, while visual servoing techniques are usually preferred for autonomous vehicle guidance in trajectory control on simple roads such as highways.

3 The vision-based behavioral architecture

The behavioral concept aims at designing simple autonomous behaviors that grouped together may perform structured tasks in real worlds. The behavioral approach is inspired to some extent by the animal world. A behavior may be described as an independent stereotyped action that is maintained by a specific perceived stimulus [21, 26].

MANO (Mobile Autonomous NOmad-200) is our implementation of the behavioral approach [3, 4]. It consists of a development and experimentation environment based on a mobile robot, dedicated vision hardware and a number of interconnected workstations. This environment offers features such as network-wide development and experimentation capabilities, virtual robot interface (allowing equivalent experimentation on simulator or real robot) and multi-language support.

Figure 3: *Layered architecture of the MANO behavioral system.*

Usually, several behaviors can be activated simultaneously, provided they are not competing for a common resource. The selection of one or more behavior is performed according to a decision scheme dictated by a planner. In the behavioral concept, planning acts on the system by allowing behaviors to run or not and is thought to handle in interaction with some model representation of the environment in form of a map (see Figure 3).

Vision-based behaviors are characterized by the fact that their stimulus is a visual primitive that triggers and maintain the behavior active as long as it exists. The vision systems we use are described in [8, 9, 22]. Examples of vision-based behaviors we developed for the MANO architecture are *going towards a landmark*, *going along a wall*, *avoiding obstacle*, *mapping*, *pushing chairs*, *homing on landmarks* and *homing on corners*. Of course, the planner also relies on behaviors based on other sensor devices such as odometers, IR sensors and sonars.

4 Characterizing homing sites in unknown environments

A homing site is a feature or a collection of features in a region of the environment that may stimulate a homing behavior. Examples of features are wall corners, retro-reflective landmarks, doors, windows and fluorescent tubes. Under the action of a homing behavior, the robot moves are directed towards a neutral configuration, which is the center of the homing site.

4.1 Homing sites in the robot configuration space

Our robot has four degrees of freedom (i.e. the turret and base can be steered independently): the parameters x and y describe the robot position on the ground plane, θ the robot base orientation and ψ the turret orientation. Let us consider a 4D hypermap of the environment, for which each element $s_i(X)$, $X = (x, y, \theta, \psi)$, takes a Boolean value representing the stimulus state of a homing behavior $\mathcal{H}_i$. More formally we have

$$s_i(X) = \begin{cases} 1 & \text{if } \mathcal{H}_i \text{ is stimulated} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The set of configuration points $S_i = \{X \mid s_i(X) = 1\}$ represents the region of the robot space for which the homing behavior $\mathcal{H}_i$ will be stimulated. We call this region the *capture zone*. The size and shape of the envelope bounding S_i partly depends on fixed parameters of the homing behavior (for example the sensor field of view), but may also vary for different homing sites. Factors such as the quality and the structure of the environment feature(s) creating the site can reduce, but not increase, S_i.

The center of the homing site towards which the robot moves are directed is denoted by C_i. Note that C_i is not the center of mass of S_i, but depends on the constraints that are applied to the visual primitives for one particular homing behavior.

Figure 4 shows an example of capture zone modelled for a homing behavior based on a range-finder vision sensor. The visual primitive is the corner formed by the intersection of the two perpendicular walls and the site center C_i corresponds to the origin of the co-ordinate system. As shown in the example, the size of the capture zone in the robot configuration space is usually limited by obstacles in the environment (in this case the corner site itself). The shape and size of the capture zone may vary considerably for other homing behaviors, but is usually a closed surface.

Cases may arise for which the visual primitives are hidden by an obstacle for a subset of S_i, resulting in

Figure 4: *example of a capture zone in the robot 3D configuration space. For sake of simplicity, the turret orientation is constrained here to $\psi = \theta$ and the robot is a single 0-dimensional point.*

one or more holes on the capture zone surface. Besides, degenerated cases with more than one site center C_i are possible.

Normally, when the homing is finished, the robot does not end up exactly at C_i, but rather in an uncertainty region centered on C_i, which size is much smaller than that of the capture zone. Various approaches may be used to model this uncertainty region [1, 5, 6]. Their application is however outside the scope of this paper.

4.2 Site learning and validation

An important issue in a behavioral architecture is stability: behaviors that may drive the robot in unstable situations are not suitable for autonomous navigation. Stability depends partly on the homing behavior and partly on the nature of the homing site, which may have characteristics that differ slightly from the ideal site the homing behavior was programmed for. Hence, for a fixed homing behavior, the structure of the environment in terms of homing sites has to be characterized.

The robot may learn new homing sites by wandering in an unknown environment and monitoring the homing behaviors stimuli. When the robot is trapped in the capture zone of a homing site candidate, instead of moving directly towards the site center, it follows a specific pattern of moves that are measured locally (for example by the odometric sensors) along the capture zone boundary, so that part of the set of points

S_i can be estimated.

Let us denote by Ω_i the closed surface bounding S_i. The characterization of Ω_i in the configuration space from a reasonably small subset of S_i is not an easy task. Instead, we consider Ω_i', a projection of Ω_i on the ground plane that is approximated by a polyhedra. In most cases, Ω_i' contains enough information about Ω_i for the validation and navigation processes.

When a new site candidate is found, the following criteria are applied and the site accepted if one is satisfied ($\mathcal{P}$ denotes the perimeter and $\mathcal{S}$ the surface):

1. there is only one C_i

2. $s_{min} < \mathcal{S}(\Omega_i') < s_{max}$

3. $\frac{\mathcal{P}(\Omega_i)}{\mathcal{S}(\Omega_i)} < k_{max}$

The first and second criteria are self-explicit. The third criteria provides a way to determine the homogeneity of Ω_i' (smooth boundary). The constants s_{min}, s_{max} and k_{max} are fixed for a given homing behavior. If all of the criteria applies, the site candidate is assumed to preserve stability and its characteristics are stored in a map.

5   Navigation with homing sites

A particularly critical aspect for a human is the design of a structured task within a behavioral context; there is a gap between what the robot may sense in terms of behavior stimuli and how the human perceives a particular situation. Self-positioning helps in reducing this gap by providing navigation to the robot in terms most suitable to human perception. Navigating with self-positioning comes down to monitoring the robot moves between homing sites, for example by using the robot odometric sensors.

For autonomous navigation, it may be desirable to have the robot build and maintain a map of the environment by itself. An advantage of self-positioning is that it provides means for the robot to learn the structure of the environment by reducing its huge state space to a small amount of homing sites and paths. An internal representation is needed, so that the sequence of moves and homing sites may be inverted and retraced at any time. Several approaches have been proposed in the literature for map representation: geometric, probabilistic (which retain some properties of the geometric representation), occupancy grid, graph-based or topological. Among them, the graph-based representation usually mixes properties of some or all the other representations [13].

We describe the environment by a graph-map M defined by

$$M = (H, P) \qquad (2)$$

where H is a set of vertices describing the homing sites and P a set of edges describing the relative geometrical (or odometric) paths binding two sites (h_i, h_j). The graph-map M is restricted to cycles of length greater than 2, so that it does not contain redundant paths between two same sites. By keeping an edge path short, navigation can take advantage of the relative accuracy of odometric sensors on short distances (see Figure 6).

6   Implementation and experimentations

We developed three homing behaviors for evaluating the self-positioning concept on a real robot. They are all based on different vision systems and are therefore stimulated by different features of the environment, which location is not known *a priori*. Some features may however be placed intentionally in the environment (for example landmarks).

6.1   Homing on corners

The sensor used by the *homing on corners* behavior is a range-finder based on a laser-line stripping vision system [8, 22]. The behavior is stimulated when a corner is recognized in the scene (see Figure 1). The site center C_c (c stands for *corner*) lies on the corner bisecting line at a distance that is inversely proportional to the sensor depth error distribution. This parameter is fixed in the behavior and is hence similar between corner homing sites. Figure 5 shows a plot of data measured for the projection Ω_c' of the capture zone on the ground plane. The axes $0x$ and $0y$ represent the two perpendicular side walls.

6.2   Homing on landmarks

The sensor used by the *homing on landmarks* behavior is a light-projecting vision system using an omnidirectional sensor (camera with fisheye lens with about 2π steradian field of view) to distinguish reflective landmarks from the background [2, 8, 9] (see Figure 7). Self-positioning can be performed on any two distinct landmarks. The site center C_l (l stands for *landmark*) lies on a line that passes half-way between the landmarks, perpendicular to the line supporting them. The distance between the site center and the landmarks, as well as the size of Ω_l', is proportional to the distance between the landmarks themselves. Hence, homing sites consisting of landmarks

the behavior. We are currently evaluating promising techniques based on fuzzy logic.

Figure 5: *Measures of the capture zone ground projection for the* homing wall corners (Ω_c') *and* homing landmarks (Ω_l') *behaviors.*

disposed too far away may be rejected in the validation process (according to the second criteria discussed earlier).

Figure 5 shows a plot of measures of Ω_l'. The distance between the two site landmarks, as well as the site location, have been chosen so that Ω_l' and Ω_c' are approximately equivalent in surface and position. It is interesting to observe that the capture zones of the *homing on landmarks* and *homing on corners* have a similar shape, although they use very different vision sensors and site features.

6.3 Homing on ceiling structures

Since typical robot workplaces are in most cases constrained to a flat surface, a particularly interesting set up for a passive omnidirectional sensor is to place it vertically so that the optical axis is perpendicular to the ceiling. The simplicity of ceiling structures in typical office-like environments is tempting since the image processing complexity is greatly reduced. Furthermore, the *homing on ceiling structures* behavior can take advantage of interesting symmetric properties when servoing the robot.

6.4 Implementation

Until now, classic servoing controllers [17, 18] have been evaluated for the implementation of the homing behaviors. However, a common problem is speed: depending on the homing behavior and the site, the robot needs 10 to 120 seconds to reach the site center starting at the boundary of the capture zone. Better servo algorithms may reduce the time necessary for homing the robot without sacrificing the stability of

Figure 6: *The robot is moving from site 1 to site 3 along simple odometric paths and uses self-positioning to reset the odometric drift.*

6.5 Experimental results

We ran as set of experimental tests that showed great stability for the *homing on corners* and *homing on landmarks* behaviors [2, 3]. The *homing on ceiling structures* is currently being evaluated. Table 1 presents an abstract of robot position measurements (orientation is not measured) during a test bench for which the robot had to navigate between three fixed homing sites (see Figure 6) using the *homing on landmarks* behavior and the odometric sensors. The values reported in the columns represent the distance in centimeters between the ideal site centers (estimated from Ω_1', Ω_2' and Ω_3') and manually measured robot positions, for the first 24 loops. These observations show no sign of incremental drift, as it would be the case using only odometric sensors for navigation.

The *homing on landmarks* behavior has been also supporting navigation for a structured task we developed for tidying up chairs in a room [2]. This task ran successfully during demonstration sessions of about one hour, showing good autonomy (see Figure 7). The fact that the chairs end positions are stable and localized in space is to our point of view a novel feature over existing tasks that are usually limited to stimulus/response-like actions not mapped in space.

7 Conclusion

We proposed and developed three new homing behaviors providing self-positioning capabilities to au-

Sites	Loops							
#	1	2	3	4	5	6	7	8
1	0	12	8	15	13	7	13	2
2	13	17	5	2	10	8	5	11
3	16	18	16	12	15	4	2	18
#	9	10	11	12	13	14	15	16
1	12	14	5	7	12	10	14	7
2	6	10	5	11	4	12	16	10
3	11	13	9	4	10	7	12	16
#	17	18	19	20	21	22	23	24
1	14	12	8	12	5	4	11	17
2	7	6	4	11	8	7	12	7
3	16	16	13	5	10	11	9	10

Figure 7: *Behaviors such as homing, going to a target and detect obstacle are used cooperatively for tiding up chairs in a room. A light-projecting vision system is used for homing the robot on landmark pairs.*

Table 1: *drift (in centimeters) of the robot while navigating between three homing sites along odometric paths.*

tonomous mobile robots: *homing on wall corners, homing on landmarks* and *homing on ceiling structures.* Evaluations performed on a real robot and involving the two first homing behaviors showed good stability and autonomy during various test benches we ran. They also provide comfortable modularity and independency features for programming complex tasks in real environments. The third homing behavior shows great potential and is currently being evaluated.

The homing behaviors provide means for the robot to learn the structure of unknown environments by reducing their huge spatial state spaces to a small amount of homing sites and paths. With them, navigation is possible in terms that were conceptually reserved so far to positioning-like approaches (using geometric an probabilistic methods).

In the future we will extend further the self-positioning concept in a task that tidies up and moves chairs in a real world consisting of offices and hallways.

Acknowledgements

References

[1] H. Takeda, C. Facchinetti and J.-C. Latombe, "Planning the Motions of a Mobile Robot in a Sensory Uncertainty Field," to be published in IEEE Transactions on Pattern Analysis and Machine Intelligence, 1994.

[2] C. Facchinetti, "Composed behaviors: tidying up chairs in a room," technical report, ref. 355HU02/94, Institute for Microtechnology, Neuchâtel, Switzerland, January 1994.

[3] J.-P. Müller and H. Hügli, "Architecture of an autonomous system: application to mobile robot navigation," proc. of the NRP 23 - Symposium on Artificial Intelligence and Robotics, pp. 101-120, Zurich, Switzerland, October 1993.

[4] H. Hügli, F. Tièche, F. Chantemargue and G. Maître, "Architecture of an experimental vision-based robot navigation system," proc. of Swissvision, Zurich, Switzerland, September 1993.

[5] C. Facchinetti, "Motion Planning and Control With Uncertainty While Sensing the Environment," Proc. International Conference on Signal Processing Applications and Technology '93, Santa Clara (USA), September 1993.

[6] C. Durieu, J. Opderbecke and G. Allegre, "A Data Fusion Application for Location of a Mobile Robot Using Odometer and a Panoramic Laser Telemeter," Proc. of the Third Int. Conf. on Intelligent Autonomous Systems, pp 519-529, Pittsburgh, February 1993.

[7] R. Basri and E. Rivlin, "Homing Using Combinations of Model Views," Int. Joint Conference on Artifical Intelligence, September 1993

[8] H. Hügli, G. Maître, F. Tièche, C. Facchinetti, "Vision-based behaviors for robot navigation,"

Proc. on the Fourth Annual SGAICO Meeting, Neuchâtel, Switzerland, September 1992.

[9] C. Facchinetti, H. Hügli, "Two Vision-Based Behaviors for Autonomous Mobile Robots," Proc. on the First Swiss Symposium on Pattern Recognition and Computer Vision, Lausanne, January 1992.

[10] J.J. Leonard and H.F. Durrant-Whyte, "Dynamic Map Building for an Autonomous Mobile Robot," the Int. Journal of Robotics Research, Vol. 11, No. 4, August 1992.

[11] F. Chenavier and J.L. Crowley, "Position Estimation for a Mobile Robot Using Vision and Odometry," Proc. of the IEEE Conf. on Robotics and Automation, pp 2588-2593, Nice, France, 1992.

[12] O. Faugeras and Z. Zhang, "A 3D World Builder with a Mobile Robot," Int. Journal of Robotics Research, vol.11, no.4, August 1992.

[13] G. Dudek, M. Jenkin, E. Milios and D. Wilkes , "Robotic Exploration as Graph Construction," IEEE Transactions on Robotics and Automation, vol. 7, no. 6, pp. 859-865, December 1991.

[14] A. Sabater and F. Thomas, "Set Membership approach to the Propagation of Uncertain Geometric Information," Proc. of the IEEE Int. Conf. on Robotics and Automation, Sacramento, 1991.

[15] P. Allen, B Yoshimi and A. Timcenko, "Real-time visual servoing," IEEE Proc. of the Int. Conf. on Robotics and Automation, Sacramento, pp 851-856, April 1991.

[16] Y. Kay and S. Lee, "A robust 3D motion estimation with stereo cameras on a robot manipulator," IEEE Proc. of the Int. Conf. on Robotics and Automation, Sacramento, pp 1102-1107, April 1991.

[17] R. Pissard-Gibollet and P. Rives, "Asservissement visuel appliqué à un robot mobile: état de lart et modélisation cinématique" INRIA, Rapport de Recherche No 1577, 1991.

[18] F. Chaumette, P. Rives and B. Espiau, "Positioning of a robot with respect to an object, tracking it and estimating its velocity by visual servoing," IEEE Proc. of the Int. Conf. on Robotics and Automation, Sacramento, pp 2248-2253, April 1991.

[19] J. Feddema and O. Mitchell, "Weighted selection of image features for resolved rate visual feedback control," IEEE trans. on Robotics and Automation, vol7, n1, pp 31-47, February 1991.

[20] D. Vernon and M. Tistarelli, "Using camera motion to estimate range for robotic manipulation," IEEE trans. on Robotics and Automation, vol6, n5, pp 509-521, October 1990.

[21] R. C. Arkin, "Integrating Behavioral, Perceptual and World Knowledge in Reactive Navigation," Robotics and Autonomous Systems, North-Holland, pp 105-122, 1990.

[22] H. Hügli and G. Maître, "3D by structure light: implementation and evaluation of a vision system for small parts," in Optical 3-D Measurement Techniques, Ed. A. Gruen & H. Kahmen, Wien, Wichmann, pp. 468-477, 1989.

[23] J. Feddema and O. Mitchell, "Vision-guided servoing with feature-based trajectory generation," IEEE trans. on Robotics and Automation, vol5, n5, pp 691-700, October 1989.

[24] R. Deller, "Set Membership Identification in Digital Signal Processing," IEEE ASSP Magazine, (pp 4-20), October 1989.

[25] L.E. Weiss, A.C. Sanderson and C.P. Neuman, "Dynamic sensor-based control of robots with visual feedback," IEEE Journal of Robotics and Automation, vol RA-3, n5, pp 404-417, October 1987.

[26] R.A. Brooks, "A Robust Layered Control System for a Mobile Robot System," IEEE Journal of Robotics and Automation, Vol. RA-2, No 1, pp 14-13, March 1986.

[27] Kailath, "Lectures on Wiener and Kalman Filtering," Springer-Verlag 1981.