Inequality: Ordinal	
Cowell, Flachaire	
Motivation Basic Problem Previous work	Measuring Inequality with ordinal data
Approach Model Basic structure	
Characterisation Inequality	Frank Cowell ¹ Emmanuel Flachaire ²
Measures Transfer principle Reference point Sensitivity	¹ STICERD London School of Economics
Normalisation Empirical aspects	² GREQAM, Marseille.
Implementation Performance Application	Neuchâtel, June 2012
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- Inequality analysis: origins go back to Pigou and Dalton
 - explicitly tied into welfare: contrast Gini and Lorenz
 seen as more fundamental than approaches such as Pareto
- But all of this is erected on rather demanding informational structure
 - income, wealth, cardinally measurable and comparableincome, earnings usually assumed to be non-negative

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- income, earnings usually assumed to be non-negative
- Maybe need a new approach to inequality measurement

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• 3 ingredients of the income-inequality measurement problem:

• the definition of "income"

• the definition of the "income-receiving unit"

• method of aggregation

• Same issues arise in cases where "income" is ordinal

• Look at standard income-inequality problem before modelling ordinal-data problem

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• 3 ingredients:

- **"income":** family income, earnings, wealth $x \in X \subseteq \mathbb{R}$.
- "income-receiving unit": n persons
- method of aggregation: function $X^n \to \mathbb{R}$

• Usually work with $X^n_{\mu} \subset \mathbb{R}$

• X^n_{μ} : Distributions obtainable from a given total income $n\mu$ using lump-sum transfers

• Obviously can't do that here: μ is undefined

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• 3 ingredients:

- **"income":** family income, earnings, wealth $x \in X \subseteq \mathbb{R}$.
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Inequality: Ordinal

Cowell, Flachaire

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• 3 ingredients:

• "income": u = U(x).

• "income-receiving unit": *n* persons (as before)

• **method of aggregation:** function $\mathbb{U}^n \to \mathbb{R}$

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- But just assuming cardinal utility is no use
 - Already pointed out in Atkinson (1970)
 - Dalton (1920) suggested inequality of (cardinal) utility
 - But if, for all *i*, you multiply u_i by $\lambda \in (0, 1)$ and add $\delta = \mu [1 \lambda] \dots$
 - ...this will automatically reduce measured inequality.
- Is this just a technicality?
- Can we proceed just as with regular income?

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- Atkinson and Dalton examples of "aggregation process"
 - How social values are introduced into an inequality-evaluation of income distribution...
 - ...not the inequality-evaluation of a distribution of utilities.
- Sometimes these are equivalent
 - but sometimes not
 - maybe utility has no natural income equivalent?
- Case 1. U depends on x with no agreed monetary valuation
 - quality of life
 - happiness
- Case 2. *U* depends on *x* that is categorical:
 - health status
 - level of completed education
 - access to public services

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Categorical variable Example: Access to Services

Inequality: Ordinal			
Cowell, Flachaire		Case 1	Case 2
Motivation		n_k	n_k
Basic Problem Previous work	Both Gas and Electricity	25	0
Approach	Electricity only	25	50
Model Basic structure Characterisation	$\underline{\underline{\mathbf{G}}}$ as only	25	50
Inequality Measures	Neither	25	0
Transfer principle			

• Suppose we have no information about needs / usage

• Nevertheless it is clear that Case 1 seems more unequal than Case 2

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Categorical variable Example: Access to Services

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Basic Problem		n_k	n_k
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Summary

• We could try to develop dominance criteria based on median

• Median may be well defined although mean is not

• what principle should play the role that is played by PoT in income inequality?

- Could try a family of measures using only median
- For such things as happiness could just use arbitrary cardinalisation
 - over large part of domain may be empirically robust
 - psychologists think Likert scales are OK for cardinalising
 - but what happens in tails?

Ways Forward?

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Summary

Step 1 is to define status

- depends on the purpose of inequality analysis
- depends on structure of information
- conventional inequality approach only works in narrowly defined information structure
- In some cases a person's status is self-defining
 - income
 - wealth
- In some cases status is defined given additional distribution-free information
 - example: if it is known that utility is $\log(x)$
- In some cases status requires information dependent on distribution
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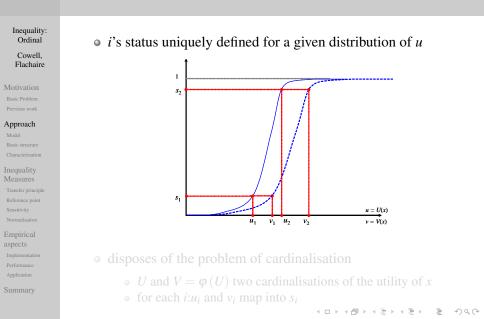
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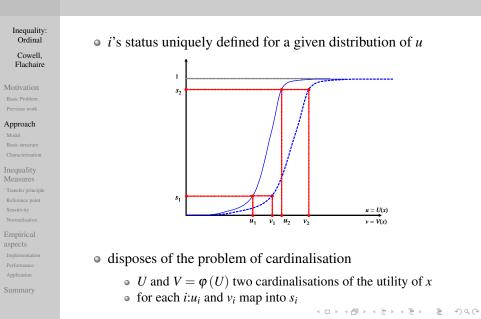
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• This approach works for categorical data

- we just have an ordered arrangement of categories 1, 2, ..., k, ..., K
- and the numbers in each category $n_1, n_2, ..., n_k, ..., n_K$

• Merger principle

- merge two adjacent categories that are irrelevant for i
- then this should leave *i*'s status unaltered
- Merger principle implies that s should be additive in the n_k

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• individual's status is given by $s \in S \subseteq \mathbb{R}$

• status determined from utility using ψ

• vector of status in a population of size $n : \mathbf{s} \in S^n$.

 $e \in S$: an equality-reference point

- could be specified exogenously
- could also depend on status vector $e = \eta$ (s)
- η need not be increasing in each component of s
- Inequality: aggregate distance from *e*
 - don't need an explicit distance function
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• [Continuity] \succeq is continuous on S^n .

- [Monotonicity in distance] If $\mathbf{s}, \mathbf{s}' \in S_e^n$ differ only in their *i*th component then (a) if $s'_i \ge e : s_i > s'_i \iff \mathbf{s} \succ \mathbf{s}'$; (b) if $s'_i \le e : s'_i > s_i \iff \mathbf{s} \succ \mathbf{s}'$.
- **[Independence]** For $\mathbf{s}, \mathbf{s}' \in S_e^n$, if $\mathbf{s} \sim \mathbf{s}'$ and $s_i = s'_i$ for some i then $\mathbf{s}(\varsigma, i) \sim \mathbf{s}'(\varsigma, i)$ for all $\varsigma \in [s_{i-1}, s_{i+1}] \cap [s'_{i-1}, s'_{i+1}]$.
- [Anonymity] For all $s \in S^n$ and permutation matrix P, Ps $\sim s$

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Standard result

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Summary

Theorem

Continuity, Monotonicity, Independence, Anonymity jointly imply \succeq is representable by the continuous function $I: S_e^n \to \mathbb{R}$ where $I(\mathbf{s}; e) = \Phi(\sum_{i=1}^n d(s_i, e), e)$, where $d: S \to \mathbb{R}$ is a continuous function that is strictly increasing (decreasing) in its first argument if $s_i > e(s_i < e)$.

Corollary

Inequality is total "distance" from equality. Distance d is continuous, satisfies d(e,e) = 0. d(s,e) is increasing in status if you move away from the reference point.

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- We need to impose more structure on the problem
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Summary

• We now impose yet more structure on the problem

• **[Ratio scale irrelevance]** Suppose there are $\mathbf{s} \in S_e^n$ and $\mathbf{s}^\circ \in S_{e^\circ}^n$ such that $\mathbf{s} \sim \mathbf{s}^\circ$. Then for all $\lambda > 0$, $\mathbf{s}' \in S_{e'}^n$ and $\mathbf{s}^\circ \in S_{e'}^n$ such that for each $i, s'_i/e = \lambda s_i/e$ and

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Inequality: Ordinal Cowell, Flachaire

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Summary

- We now impose yet more structure on the problem
- **[Ratio scale irrelevance]** Suppose there are $\mathbf{s} \in S_e^n$ and $\mathbf{s}^{\circ} \in S_{e^{\circ}}^n$ such that $\mathbf{s} \sim \mathbf{s}^{\circ}$. Then for all $\lambda > 0$, $\mathbf{s}' \in S_{e'}^n$ and $\mathbf{s}'' \in S_{e'}^n$ such that for each $i, s'_i/e = \lambda s_i/e$ and $s_i''/e = \lambda s_i^{\circ}/e^{\circ}$: $\mathbf{s}' \sim \mathbf{s}''$.

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Inequality: Ordinal Cowell, Flachaire

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- such that $s_i < s_j$ and $s_i + \delta < s_j \delta$),
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- But is this property attractive?

Inequality: Ordinal Cowell, Flachaire

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Four distributional scenarios (1)

Inequality: Ordinal Cowell, Flachaire		Case 0	Cas	se 1	Cas	se 2	Ca	ise 3
Motivation Basic Problem Previous work Approach Model Basic structure Characterisation	B E G N	$ \begin{array}{cccc} n_k & s_i \\ 0 & & \\ 50 & 1 \\ 25 & \frac{1}{2} \\ 25 & \frac{1}{4} \end{array} $		S_i 1 3/4 1/2 1/4	$egin{array}{c} n_k \ 0 \ 50 \ 50 \ 0 \end{array}$	s_i $\frac{1}{1/2}$	n_k 25 25 50 0	s_i 1 3/4 1/2
Inequality Measures Transfer principle Reference point Sensitivity Normalisation Empirical	$\mu(\mathbf{s})$	11/1 # persons in		5/8 $k \in \{$	B E G	3/4 N}		11/16
Application		$\sum_{\ell=1}^{k(i)} n_\ell - $						

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Four distributional scenarios (1)

Inequality: Ordinal									
Cowell, Flachaire		Ca	se 0	Cas	se 1	Cas	se 2	Ca	se 3
Motivation		n_k	Si	n_k	Si	n_k	Si	n_k	Si
Basic Problem	В	0		25	1	0		25	1
Previous work	Е	50	1	25	3/4	50	1	25	3/4
Approach	G	25	1/2	25	1/2	50	1/2	50	
Basic structure							1/2	_	1/2
Characterisation	N	25	1/4	25	1/4	0		0	
Inequality Measures									
Transfer principle	$\mu(\mathbf{s})$		11/16		5/8		3/4		11/16
Reference point	• ()		/		'		/		,
Sensitivity Normalisation									
Empirical	• n_k is	# perso	ons in ca	ategory	$k \in \{\mathbf{E}$	B, E, G,	N}		

• $s_i = \frac{1}{n} \sum_{\ell=1}^{k(i)} n_\ell - downward$ -looking status

Summary

Four distributional scenarios (1)

Inequality: Ordinal									
Cowell, Flachaire		Ca	ise 0	Cas	se 1	Cas	se 2	Ca	ise 3
Motivation		n_k	Si	n_k	Si	n_k	Si	n_k	Si
Basic Problem	В	0		25	1	0		25	1
Previous work Approach	Ε	50	1	25	3/4	50	1	25	3/4
Model	G	25	1/2	25	1/2	50	1/2	50	1/2
Basic structure Characterisation	Ν	25	1/4	25	1/4	0	7	0	/
Inequality Measures			-						
Transfer principle	$\mu(\mathbf{s})$		11/16		5/8		3/4		11/16
Reference point Sensitivity	• ()		,		/		,		,
Normalisation									
Empirical aspects	• n_k is	# pers	ons in ca	ategory	$k \in \{\mathbf{E}\}$	B, E, G,	N		

• $s_i = \frac{1}{n} \sum_{\ell=1}^{k(i)} n_\ell$ - *downward*-looking status

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Four distributional scenarios

Inequality: Ordinal Cowell,		G	0	G		G	2	G	2
Flachaire		Ca	se 0	Cas	se I	Cas	se 2	Ca	ise 3
Motivation Basic Problem Previous work	В	n_k 0	s'_i	$\frac{n_k}{25}$	$\frac{s'_i}{1/4}$	n_k 0	s'_i	$\frac{n_k}{25}$	s'_i 1/4
Approach Model Basic structure Characterisation	E G N	50 25 25	$\frac{1/2}{3/4}$	25 25 25	$\frac{1/2}{3/4}$	50 50 0	$\frac{1}{2}$	25 50 0	$\frac{1}{2}$
Inequality Measures Transfer principle Reference point Sensitivity Normalisation	$\mu(\mathbf{s})$		11/16		5/8		3/4		11/16
Empirical aspects Implementation	• n_k is	<u>^</u>	ons in ca	ategory	$k \in \{\mathbf{I}\}$	B,E,G,	N}		

- Performance
- Application

Summary

 $s'_i = \frac{1}{n} \sum_{\ell=k(i)}^{K} n_{\ell} - upward$ -looking status

Four distributional scenarios

Inequality: Ordinal									
Cowell, Flachaire		Ca	se 0	Cas	se 1	Cas	se 2	Ca	se 3
Motivation		n_k	s'_i	n_k	s'_i	n_k	s'_i	n_k	s'_i
Basic Problem	В	0		25	1/4	0		25	1/4
Previous work Approach	Е	50	1/2	25	1/2	50	1/2	25	1/2
Model	G	25	3/4	25	3/4	50	1	50	1
Basic structure Characterisation	Ν	25	1	25	1	0		0	
Inequality Measures									
Transfer principle	$\mu(\mathbf{s})$		11/16		5/8		3/4		11/16
Reference point Sensitivity	,		•						

• n_k is # persons in category $k \in \{B, E, G, N\}$

• $s'_i = \frac{1}{n} \sum_{\ell=k(i)}^{K} n_{\ell}$ - *upward*-looking status

Summary

Four distributional scenarios

Inequality: Ordinal									
Cowell, Flachaire		Cas	se 0	Cas	se 1	Cas	se 2	Ca	se 3
Motivation		n_k	s'_i	n_k	s'_i	n_k	s'_i	n_k	s'_i
Basic Problem	В	0		25	1/4	0		25	1/4
Previous work Approach	E	50	1/2	25	1/2	50	1/2	25	1/2
Model	G	25	3/4	25	3/4	50	1	50	1
Basic structure Characterisation	N	25	1	25	1	0		0	
Inequality Measures									
Transfer principle	$\mu(\mathbf{s})$		11/16		5/8		3/4		11/16
Reference point Sensitivity	. ()		,		,		1		,

• n_k is # persons in category $k \in \{B, E, G, N\}$

•
$$s'_i = \frac{1}{n} \sum_{\ell=k(i)}^{K} n_{\ell}$$
- *upward*-looking status

Summary

Four distributional scenarios (2)

Inequality: Ordinal									
Cowell, Flachaire		Ca	se 0	Cas	se 1	Cas	se 2	Ca	ise 3
		n_k	S_i	n_k	S_i	n_k	S_i	n_k	S_i
Motivation	В	0		25	1	0		25	1
Basic Problem	D	0		25	1	0		25	1
Previous work	E	50	1	25	3/4	50	1	25	3/4
Approach	G	25	1/2	25	1/2	50	1/2	50	1/2
Model	G	25	1/2	23	1/2	50	1/2	50	1/2
Basic structure	Ν	25	1/4	25	1/4	0		0	
Characterisation	1	25	-/4	25	-/4	0		0	
Inequality									
Measures	u (a)		11/16		5/8		3/4		11/16
Transfer principle	$\mu(\mathbf{s})$		11/16		5/8		3/4		11/16
Pafaranca point									

• Case 0 to Case 1:

- 25 people promoted from E to B
- if *e* equals to any of values taken by $\mu(s)$
- then inequality increases

Four distributional scenarios (2)

Inequality: Ordinal									
Cowell, Flachaire		Ca	se 0	Cas	se 1	Cas	se 2	Ca	se 3
		n_k	S_i	n_k	S_i	n_k	Si	n_k	Si
Motivation Basic Problem	В	0		25	1	0		25	1
Previous work	Ε	50	1	25	3/4	50	1	25	3/4
Approach	G	25	1/2	25	1/2	50	1/2	50	1/2
Basic structure Characterisation	Ν	25	1/4	25	1/4	0		0	
Inequality									
Measures Transfer principle	$\mu(\mathbf{s})$		11/16		5/8		3/4		11/16
Reference point Sensitivity									
Normalisation	• Case	0 to C	ase 1:						
The state of the set	- Cube	0.000							

- 25 people promoted from E to B
- if *e* equals to any of values taken by $\mu(\mathbf{s})$

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• then inequality increases

Four distributional scenarios (3)

Inequality: Ordinal									
Cowell, Flachaire		Ca	se 0	Cas	se 1	Cas	se 2	Ca	se 3
		n_k	S_i	n_k	S_i	n_k	S_i	n_k	S_i
Motivation	В	0		25	1	0		25	1
Basic Problem	D	0		25	1	0		25	1
Previous work	E	50	1	25	3/4	50	1	25	3/4
Approach	G	25	1/2	25	1/2	50	1/2	50	1/2
Model	G	23	1/2	23	1/2	50	1/2	50	1/2
Basic structure	Ν	25	1/4	25	1/4	0		0	
Characterisation	14	23	-/+	23	-/+	0		0	
Inequality									
Measures	$u(\mathbf{s})$		11/16		5/8		3/4		11/16
Transfer principle	$\mu(\mathbf{s})$		11/16		5/8		5/4		11/10

• Case 0 to Case 2:

- 25 people promoted from N to G
- if *e* equals to any of values taken by $\mu(\mathbf{s})$
- then inequality decreases

Four distributional scenarios (3)

Inequality: Ordinal									
Cowell, Flachaire		Ca	se 0	Ca	se 1	Cas	se 2	Ca	se 3
		n_k	Si	n_k	Si	n_k	Si	n_k	Si
Motivation Basic Problem	В	0		25	1	0		25	1
Previous work	Е	50	1	25	3/4	50	1	25	3/4
Approach Model	G	25	1/2	25	1/2	50	1/2	50	1/2
Basic structure Characterisation	Ν	25	1/4	25	1/4	0		0	
Inequality Measures	u(a)		11/16		5/8		3/4		11/16
Transfer principle	$\mu(\mathbf{s})$		11/10		5/8		5/4		11/10
Reference point Sensitivity									
Normalisation	• Case	0 to C	ase 2.						
Empirical	J Case	0100	ube 2.						

- 25 people promoted from N to G
- if *e* equals to any of values taken by $\mu(\mathbf{s})$

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• then inequality decreases

Transfer Principle again

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Inequality: Ordinal									
Cowell, Flachaire		Са	ise 0	Ca	se 1	Cas	se 2	Са	se 3
Motivation		n_k	Si	n_k	S_i	n_k	Si	n_k	S_i
Basic Problem	В	0		25	1	0		25	1
Previous work	E	50	1	25	3/4	50	1	25	3/4
Approach Model	G	25	1/2	25	1/2	50	1/2	50	1/2
Basic structure Characterisation	Ν	25	1/4	25	1/4	0		0	
Inequality Measures					5/0				
Transfer principle Reference point	$\mu(\mathbf{s})$				5/8		3/4		11/1
Sensitivity									
Normalisation	• Case	e 0 to C	ase 1: ir	nequali	tv incr	eases			
Empirical aspects				<u>_</u>	~				
Implementation	• Case	e U to C	ase 2: ir	requali	ty decr	eases			
Performance Application	• Case	e 0 to C	ase 3: c	ombina	tion re	sults in	ambig	puous c	hang
S								5	

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Transfer Principle again

Inequality: Ordinal									
Cowell, Flachaire		Ca	se 0	Cas	se 1	Cas	se 2	Ca	se 3
Motivation		n_k	Si	n_k	Si	n_k	Si	n_k	Si
Basic Problem	В	0		25	1	0		25	1
Previous work	Ε	50	1	25	3/4	50	1	25	3/4
Approach Model	G	25	1/2	25	1/2	50	1/2	50	1/2
Basic structure Characterisation	N	25	1/4	25	1/4	0	/	0	7
Inequality Measures					,				
Transfer principle	$\mu(\mathbf{s})$		11/16		5/8		3/4		11/16
Reference point Sensitivity Normalisation	• ()		7		,		,		1

Empirical aspects Implementation

Application

Summary

- Case 0 to Case 1: inequality increases
- Case 0 to Case 2: inequality decreases

• Case 0 to Case 3: combination results in ambiguous change

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Transfer Principle again

Inequality: Ordinal									
Cowell, Flachaire		Case 0		Case 1		Case 2		Case 3	
Motivation		n_k	Si	n_k	Si	n_k	Si	n_k	Si
Basic Problem	В	0		25	1	0		25	1
Previous work	Ε	50	1	25	3/4	50	1	25	3/4
Approach Model	G	25	1/2	25	1/2	50	1/2	50	1/2
Basic structure Characterisation	N	25	1/4	25	1/4	0	/ -	0	/ -
Inequality			,		,				
Measures	$u(\mathbf{s})$		11/16		5/8		3/4		11/16
Transfer principle Reference point	$\mu(\mathbf{s})$		11/10		5/8		5/4		11/10

Empirical aspects Implementatio

Performance

Application

Summary

• Case 0 to Case 1: inequality increases

• Case 0 to Case 2: inequality decreases

• Case 0 to Case 3: combination results in ambiguous change

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Transfer Principle again

Inequality: Ordinal									
Cowell, Flachaire		Ca	se 0	Cas	se 1	Cas	se 2	Ca	ise 3
Motivation		n_k	Si	n_k	Si	n_k	Si	n_k	Si
Basic Problem	В	0		25	1	0		25	1
Previous work	Е	50	1	25	3/4	50	1	25	3/4
Approach	G	25	1/2	25	1/2	50	1/2	50	1/2
Basic structure Characterisation	N	25	1/4	25	1/4	0	/ -	0	/ -
Inequality Measures					- /		21		
Transfer principle	$\mu(\mathbf{s})$		11/16		5/8		3/4		11/16
Reference point Sensitivity									
Normalisation	C	0.4.0	1	1.					
Empirical	• Case	U to C	ase 1: in	equali	ty incre	eases			

• Case 0 to Case 2: inequality decreases

• Case 0 to Case 3: combination results in ambiguous change

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Summary

Transfer Principle again

Inequality: Ordinal									
Cowell, Flachaire		Ca	se 0	Cas	se 1	Cas	se 2	Ca	se 3
Motivation		n_k	Si	n_k	Si	n_k	Si	n_k	Si
Basic Problem	В	0		25	1	0		25	1
Previous work	Ε	50	1	25	3/4	50	1	25	3/4
Approach Model	G	25	1/2	25	1/2	50	1/2	50	1/2
Basic structure Characterisation	Ν	25	1/4	25	1/4	0	/	0	7
Inequality									
Measures Transfer principle	$\mu(\mathbf{s})$		11/16		5/8		3/4		11/16
Reference point Sensitivity	/								

- Case 0 to Case 1: inequality increases
- Case 0 to Case 2: inequality decreases
- Case 0 to Case 3: combination results in ambiguous change

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Outline

Inequality: Ordinal Cowell, Flachaire

Motivation Basic Problem Previous work

Approach Model Basic structure

Inequality Measures

Transfer principle

Reference point

Normalisation

Empirical aspects Implementation

Application

Summary

Motivation

• Basics

• Previous work

- Approach
 - Model
 - Basic structure
 - Characterisation

Inequality Measures

Transfer principle

Reference point

- Sensitivity
- Normalisation
- 4 Empirical aspects
 - Implementation

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- Performance
- Application
- Summarv

Inequality: Ordinal Cowell, Flachaire

Motivation Basic Problem

Previous work

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Approach
Model
```

Characterisation

Inequality Measures

Reference point

Sensitivity Normalisation

Empirical aspects

Implementatio

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Summary

• Inequality index requires a reference point

• Mean status: $e = \eta(s) = \mu(s)$

• for continuous distributions will equal 0.5

• for categorical data, there is no counterpart to fixed-mean assumption in income-inequality analysis

• Median status:
$$e = \eta$$
 (s) = med(s)

• not well-defined

• in the example median is any value in interval M(s)

- $M(\mathbf{s}) = [1/2, 1)$ in cases 0 and 2
- $M(\mathbf{s}) = [1/2, 3/4)$ in cases 1 and 3

```
• Max status: e = 1
```

for constant *e* this is only value that makes sense
natural normalisation of index is *c* = 1: ensures *I*(1;1) = 0

Inequality: Ordinal Cowell, Flachaire

Motivation Basic Problem

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Summary

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 in cases 0 and 2

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$$M(\mathbf{s}) = [1/2, 3/4)$$
 in cases 1 and 3

• Max status: e = 1

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Inequality: Ordinal Cowell, Flachaire

Motivation Basic Problem

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Inequality: Ordinal Cowell, Flachaire

Motivation Basic Problem

Approach Model Basic structur

Characterisation

Inequality Measures Transfer princip Reference point

Sensitivity

Empirical aspects

Performance

Application

Summary

• Inequality index requires a reference point

• Mean status: $e = \eta (\mathbf{s}) = \mu(\mathbf{s})$

• for continuous distributions will equal 0.5

• for categorical data, there is no counterpart to fixed-mean assumption in income-inequality analysis

• Median status:
$$e = \eta$$
 (s) = med(s)

- not well-defined
- in the example median is any value in interval $M(\mathbf{s})$
- $M(\mathbf{s}) = [1/2, 1)$ in cases 0 and 2
- $M(\mathbf{s}) = [1/2, 3/4)$ in cases 1 and 3

• Max status: e = 1

- for constant *e* this is only value that makes sense
- natural normalisation of index is c = 1: ensures I(1; 1) = 0

Outline

Inequality: Ordinal Cowell, Flachaire

Motivation Basic Problem Previous work

Approach Model Basic structure

Characterisation

Measures Transfer princip

Reference poin

Sensitivity

Normalisation

Empirical aspects Implementation Performance

Application

Summary

- Motivation
 - Basics
 - Previous work
- Approach
 - Model
 - Basic structure
 - Characterisation

Inequality Measures

- Transfer principle
- Reference point

Sensitivity

- Normalisation
- 4 Empirical aspects
 - Implementation
 - Performance
 - Application
- Summarv

Inequality: Ordinal Cowell, Flachaire

- Motivation Basic Problem
- Approach Model Basic structure
- Characterisation
- Inequality Measures Transfer princip
- Sensitivity

Normalisation

- Empirical aspects Implementatio Performance
- Application
- Summary

• α captures the sensitivity of measured inequality

• If α is high $I_{\alpha}(\mathbf{s}; e) = \frac{1}{\alpha[\alpha-1]} \left[\frac{1}{n} \sum_{i=1}^{n} \left[\frac{s_i}{e} \right]^{\alpha} - c \right]$ sensitive to high status-inequality

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• If
$$e = \mu(\mathbf{s})$$
 and $\alpha = c = 1$ then we have $I_1(\mathbf{s}; e) = \frac{1}{n} \sum_{i=1}^n \frac{s_i}{e} \log\left(\frac{s_i}{e}\right)$

Inequality: Ordinal Cowell, Flachaire

- Motivation Basic Problem
- Approach Model Basic structure
- Characterisation
- Inequality Measures Transfer princip Reference point
- Sensitivity
- Normalisation
- Empirical aspects Implementatio Performance
- Application
- Summary

- α captures the sensitivity of measured inequality
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• If
$$e = \mu(\mathbf{s})$$
 and $\alpha = c = 1$ then we have $I_1(\mathbf{s}; e) = \frac{1}{n} \sum_{i=1}^n \frac{s_i}{e} \log\left(\frac{s_i}{e}\right)$

Inequality: Ordinal Cowell, Flachaire

- Motivation Basic Problem
- Approach Model Basic structure
- Characterisation
- Inequality Measures Transfer princip Reference point
- Sensitivity
- Normalisation
- Empirical aspects Implementatio Performance
- Application
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Outline

Inequality: Ordinal Cowell, Flachaire

Motivation Basic Problem Previous work

Approach Model Basic structure

Inequality Measures Transfer principl Reference point Sensitivity

Normalisation

Empirical aspects Implementation Performance Application

Summary

- Motivation
 - Basics
 - Previous work
- Approach
 - Model
 - Basic structure
 - Characterisation

Inequality Measures

- Transfer principle
- Reference point
- Sensitivity
- Normalisation
- Empirical aspects
 - Implementation

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- Performance
- Application
- Summarv

Inequality: Ordinal					
Cowell,		Case 0	Case 1	Case 2	Case 3
Flachaire	$\mu(\mathbf{s})$	11/16	5/8	3/4	11/16
Motivation	$med_1(s)$	3/4	5/8	3/4	5/8
Basic Problem Previous work	$med_2(\mathbf{s})$	1/2	1/2	1/2	1/2
Approach	$I_0(\mathbf{s}; \boldsymbol{\mu}(\mathbf{s}))$	0.1451	0.1217	0.0588	0.0438
Model Basic structure	$I_0(\mathbf{s}; \operatorname{med}_1(\mathbf{s}))$	0.2321	0.1217	0.0588	-0.0515
Characterisation	$I_0(\mathbf{s}; \operatorname{med}_2(\mathbf{s}))$	-0.1732	-0.1013	-0.3465	-0.2746
Measures	$I_0(\mathbf{s}; 1)$	0.5198	0.5917	0.3465	0.4184
Transfer principle Reference point	10(0, 1)	0.5170	0.0917	0.5 105	0.1101
Sensitivity	• $I_0(\mathbf{s}; \boldsymbol{\mu}(\mathbf{s})),$	$I_0(\mathbf{s}; \text{med}_1)$	(s)): inequalit	v decreases v	when one
Normalisation				<i>j meerenses</i> (
Empirical	person prom	noted from I	E to B		
aspects	• Case 0 t	to Case1, or (Case 2 to Case	3	

- movement changes both the $\mu(s)$ and med₁(s) ref points
- $I_0(\mathbf{s}; \operatorname{med}_2(\mathbf{s})) < 0$ for *all* cases in example!

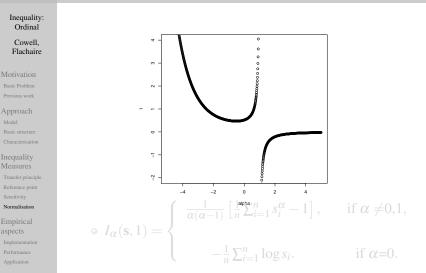
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Approach	$I_0(\mathbf{s}; \boldsymbol{\mu}(\mathbf{s}))$	0.1451	0.1217	0.0588	0.0438
Basic structure	$I_0(\mathbf{s}; \operatorname{med}_1(\mathbf{s}))$	0.2321	0.1217	0.0588	-0.0515
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Measures Transfer principle	$I_0(\mathbf{s}; 1)$	0.5198	0.5917	0.3465	0.4184
Reference point Sensitivity Normalisation Empirical aspects Implementation Performance		noted from E to Case1, or C	E to B Case 2 to Case	-	
Application		•	• • • /		I · ···
Summary	• $I_0(\mathbf{s}; \operatorname{med}_2(\mathbf{s}))$	(5)) < 0 for a	ill cases in ex	ample!	
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aspects	• Case 0 t	o Case1, or (Case 2 to Case	3	
Implementation Performance Application	• moveme	ent changes b	both the $\mu(\mathbf{s})$ a	nd $med_1(\mathbf{s})$ res	f points
Summary	• $I_0(\mathbf{s}; \operatorname{med}_2(\mathbf{s}))$	(s)) < 0 for a	all cases in ex	ample!	

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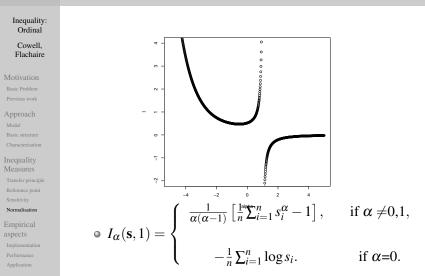
Behaviour of $I_{\alpha}(\mathbf{s}; 1)$ with α



Summary

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Behaviour of $I_{\alpha}(\mathbf{s}; 1)$ with α



Summary

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Inequality: Ordinal Cowell, Flachaire

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• Inequality can also be written $I_{\alpha}(\mathbf{s}, 1) = \frac{1}{\alpha - 1} \left| \frac{1}{n} \sum_{i=1}^{n} \frac{s_{i}^{\alpha} - 1}{\alpha} \right|$

• if
$$0 < s < 1$$
 then $[s^{\alpha} - 1]/\alpha < 0$ and if $s = 1$ then $[s^{\alpha} - 1]/\alpha = 0$

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Implementation

Description of sample

 $x_i = \begin{cases} 1 & \text{with sample proportion } p_1 \\ 2 & \text{with sample proportion } p_2 \\ \dots \\ K & \text{with sample proportion } p_K \end{cases}$

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Point estimate of the index:

• $I_{\alpha} = \begin{cases} \frac{1}{\alpha(\alpha-1)} \left[\sum_{i=1}^{K} p_i \left[\sum_{j=1}^{i} p_j \right]^{\alpha} - 1 \right] & \text{if } \alpha \neq 0, 1 \\ \\ -\sum_{i=1}^{K} p_i \log \left[\sum_{j=1}^{i} p_j \right] & \text{if } \alpha = 0 \end{cases}$ • function of *K* parameter estimates (p_1, p_2, \dots, p_K) following a

multinomial

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Estimator of cov matrix of (p_1, p_2, \dots, p_k) is $\Sigma = \frac{1}{n} \begin{bmatrix} p_1(1-p_1) & -p_1p_2 & \dots & -p_1p_K \\ -p_2p_1 & p_2(1-p_2) & \dots & -p_2p_K \\ \vdots & \vdots & \vdots & \vdots \\ -p_Kp_1 & -p_Kp_2 & \dots & p_K(1-p_K) \end{bmatrix}$

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• From the CLT I_{α} is asymptotically Normally distributed

Estimator of cov matrix of (p_1, p_2, \dots, p_k) is $\Sigma = \frac{1}{n} \begin{bmatrix} p_1(1-p_1) & -p_1p_2 & \dots & -p_1p_K \\ -p_2p_1 & p_2(1-p_2) & \dots & -p_2p_K \\ \vdots & \vdots & \vdots & \vdots \\ -p_Kp_1 & -p_Kp_2 & \dots & p_K(1-p_K) \end{bmatrix}$

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Summary

• 3 variants of CIs: <u>Asymptotic</u>, <u>Percentile</u> Bootstrap, Studentized Bootstrap

• $CI_{asym} = [I_{\alpha} - c_{0.975} \widehat{\operatorname{Var}}(I_{\alpha})^{1/2}; I_{\alpha} + c_{0.975} \widehat{\operatorname{Var}}(I_{\alpha})^{1/2}]$

• $c_{0.975}$ from the Student distribution T(n-1)

• do not always perform well in finite samples

• Bootstraps: generate resamples, $b = 1, \dots, B$

- for each resample *b* compute the inequality index
- obtain *B* bootstrap statistics, I_{α}^{b}

• also *B* bootstrap *t*-statistics $t^b_{\alpha} = (I^b_{\alpha} - I_{\alpha})/\widehat{\operatorname{Var}}(I^b_{\alpha})^{1/2}$

•
$$CI_{stud} = [I_{\alpha} - c^*_{0.975} \widehat{\operatorname{Var}}(I_{\alpha})^{1/2}; I_{\alpha} - c^*_{0.025} \widehat{\operatorname{Var}}(I_{\alpha})^{1/2}]$$

• $c^*_{0.025}$ and $c^*_{0.975}$ are from EDF of the bootstrap *t*-statistics

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• also *B* bootstrap *t*-statistics $t^b_{\alpha} = (I^b_{\alpha} - I_{\alpha})/\widehat{\operatorname{Var}}(I^b_{\alpha})^{1/2}$

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$$CI_{stud} = [I_{\alpha} - c_{0.975}^* \widehat{\operatorname{Var}}(I_{\alpha})^{1/2}; I_{\alpha} - c_{0.025}^* \widehat{\operatorname{Var}}(I_{\alpha})^{1/2}]$$

• $c_{0.025}^*$ and $c_{0.975}^*$ are from EDF of the bootstrap *t*-statistic

Inequality: Ordinal Cowell, Flachaire

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Summary

• 3 variants of CIs: <u>Asymptotic</u>, <u>Percentile</u> Bootstrap, Studentized Bootstrap

• $\overline{CI_{asym}} = [I_{\alpha} - c_{0.975} \widehat{Var}(I_{\alpha})^{1/2}; I_{\alpha} + c_{0.975} \widehat{Var}(I_{\alpha})^{1/2}]$

• $c_{0.975}$ from the Student distribution T(n-1)

• do not always perform well in finite samples

• Bootstraps: generate resamples, $b = 1, \dots, B$

- for each resample *b* compute the inequality index
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• $CI_{perc} = [c_{0.025}^b; c_{0.975}^b]$ • $c_{0.025}^b$ and $c_{0.975}^b$ are from EDF of bootstrap statistics

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Confidence Intervals

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Summary

• Take an example with 3 ordered categories (K = 3)

• Samples are drawn from a multinomial distribution with probabilities $\pi = (0.3, 0.5, 0.2)$

• Is asymptotic or bootstrap distribution a good approximation of the exact distribution of the statistic?

• if we are using 95% CIs of I_{α}

• coverage error rate should be close to nominal rate, 0.05

• Check coverage error rate of CIs as sample size increases

• $\alpha = -1, 0, 0.5, 0.99$

- 199 bootstraps
- 10 000 replications to compute error rates
- n = 20, 50, 100, 200, 500, 1000

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Inequality:

Performance

Ordinal						
Cowell, Flachaire		α	-1	0	0.5	0.99
Flachane	Asymptotic B	n = 20	0.0606	0.0417	0.0598	0.0491
Motivation Basic Problem		n = 500	0.0523	0.0492	0.0521	0.0523
Previous work		n = 1000	0.0485	0.0540	0.0552	0.0549
Approach	Percentile B	n = 20	0.0384	0.0981	0.0912	0.1023
Basic structure		n = 500	0.0509	0.0513	0.0552	0.0554
Characterisation Inequality		n = 1000	0.0482	0.0556	0.0547	0.0551
Measures	Studentized B	n = 20	0.1275	0.0843	0.1041	0.1377
Transfer principle Reference point		n = 500	0.0518	0.0478	0.0429	0.0465
Sensitivity Normalisation		n = 1000	0.0473	0.0522	0.0493	0.0503
Empirical aspects	 Asymptotic 					
Implementation						

- Percentile bootstrap performs well for n > 50
 - Studentized bootstrap does not do well for small samples
- Reliable results for $\alpha = 0.99$ (index'is tindefined for $\alpha = 1 \mathcal{P}^{\alpha}$

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Outline

Inequality: Ordinal Cowell, Flachaire

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 - Characterisation
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 - Transfer principle
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 - Normalisation



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- Summary

• Life satisfaction question:

All things considered, how satisfied are you with your life as a whole these days? Using this card on which 1 means you are "completely dissatisfied" and 10 means you are "completely atisfied" where would you put your satisfaction with your life as a whole? (code one number): Completely dissatisfied – 1 2 8 4 5 6 7 8 9 10 – Completely satisfied

• Health question:

All in all, how would you describe your state of health these days? Would you say it is (read out): 1 Very good, 2 Good, 3 Fair, 4 Poor.

World values survey

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GDP and Life satisfaction

Inequality: Ordinal Cowell, Flachaire

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Summary

• Cross-country comparison of life satisfaction and GDP/head

- Easterlin or happiness-income paradox
- Weak relation internationally?

• How should we quantify life satisfaction?

- simple linearity of Likert scale from coding?
- exponential scale
- Ng (1997), Ferrer-i-Carbonell and Frijters (2004)
- Is inequality of life satisfaction related to GDP/head?

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• Use I_0 and other members of the same family

GDP and Life satisfaction

Inequality: Ordinal Cowell, Flachaire

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GDP and Life satisfaction

Inequality: Ordinal Cowell, Flachaire

Motivation Basic Problem

Approach Model Basic structure

Inequality Measures Transfer principle Reference point Sensitivity Normalisation

Empirical aspects Implementation

Application

Summary

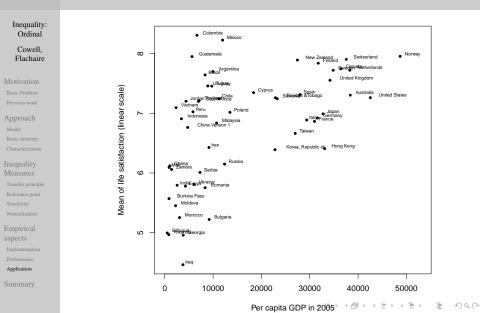
• Cross-country comparison of life satisfaction and GDP/head

- Easterlin or happiness-income paradox
- Weak relation internationally?
- How should we quantify life satisfaction?
 - simple linearity of Likert scale from coding?
 - exponential scale?
 - Ng (1997), Ferrer-i-Carbonell and Frijters (2004)
- Is inequality of life satisfaction related to GDP/head?

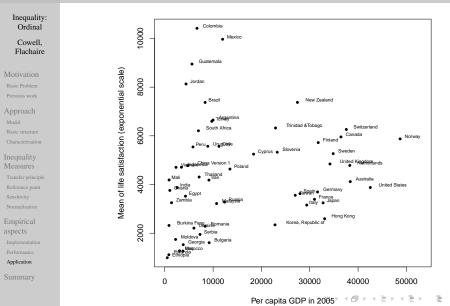
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• Use I_0 and other members of the same family

GDP and Life satisfaction (Linear)

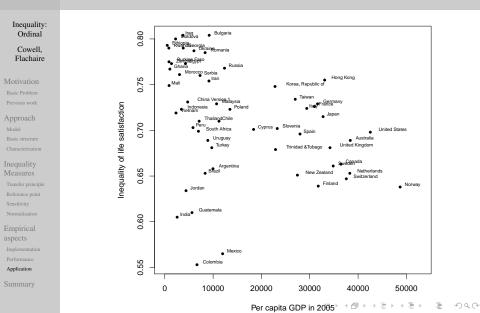


GDP and Life satisfaction (Exponential)



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GDP and Inequality of Life satisfaction



Health status

Inequality: Ordinal Cowell, Flachaire

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• use same inequality index as for life satisfaction

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Health status

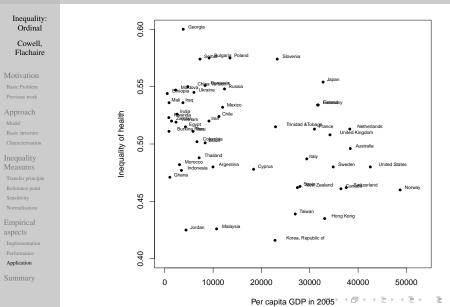
Inequality: Ordinal
Cowell, Flachaire

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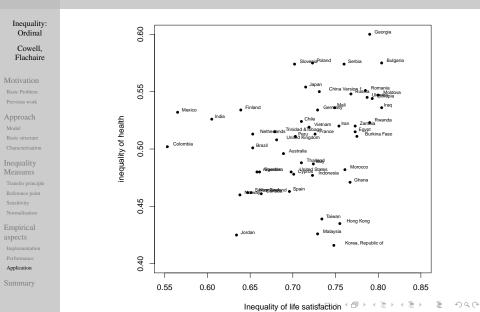
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Inequality of health and GDP



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Inequality of health



Inequality: Ordinal Cowell, Flachaire

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Summary

• Satisfaction / GDP results sensitive to the cardinal interpretation of the answers

linear: get a positive relation below \$15 000, flat after thatexponential: no relation

• OLS estimate of I_0 (life satisfaction) on the GDP per capita small and negative

• happiness-income relationship is weak in cross-country comparisons

• No clear relationship between I_0 (health) on GDP per capita

• OLS estimate of I_0 (health) on I_0 (life satisfaction) produces a slope coefficient not significantly different from zero

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• health-life satisfaction relationship is not significant

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Inequality: Ordinal Cowell. Flachaire Summary

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Inequality: Ordinal Cowell, Flachaire

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Summary

Theoretical tweaks

- alternatives concepts of status
- alternatives to scale invariance
- Interpretation in terms of inequality of opportunity

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- Further empirical applications
 - Health status
 - Education

Inequality: Ordinal Cowell, Flachaire

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Inequality: Ordinal Cowell, Flachaire

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Summary

• Inequality with ordinal data is a widespread phenomenon

• Conventional I-measures may make no sense

• Our approach:

- separates out the issue of status from that of inequality-aggregation
- allows you to choose "reference status"
- gives a family of measures
- Nice properties empirically

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Inequality: Ordinal Cowell, Flachaire

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Inequality: Ordinal Cowell, Flachaire

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