

12th International Workshop on Partial Orders in Applied
Sciences

**TOWARDS AN UNDERSTANDING OF
COMPLEX PHENOMENON: APPLYING
PARTIAL ORDER THEORY TO MULTI-
INDICATOR SYSTEM**

University of Neuchâtel

Department of Sociology

Fbg de l'Hôpital 27, 2000 Neuchâtel, Switzerland

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Session I

General role of Multi-Indicator System

**Indicators in the
framework of partial
order**

Filomena Maggino

Sapienza - University of Rome
Dep. of Statistical Sciences
Counselor of the Italian Prime Minister

Rainer Bruggemann

Leibniz Institute of Freshwater
Ecology and Inland Fisheries
Germany



(1) Introduction

(1) Introduction

TOPIC “indicators” *in science*

- considered a “niche field” in the methodological scientific debate

however

- never missed in any conference, workshop, seminar on measuring socio-economic dimensions during the last decades

(1) Introduction

TOPIC "indicators"
outside science

Everywhere indicators are used and constructed.

A look at newspapers shows the increasing importance of indicators also in the media.

(1) Introduction

TOPIC "indicators"
outside science

Often these indicators are used to rank objects, like
quality of life in cities

(1) Introduction

TOPIC “indicators”
outside science

Often these indicators are used to rank objects, like
quality of life in cities

- cities ranked with respect to “quality of life”
- regular reports of the EU, where nations are compared with respect to e.g. innovation potential
- UNICEF: child well-being
-

(1) Introduction

Indicator



what relates
concepts to reality
through observation

(1) Introduction

Indicator



not

a simply crude statistical information

but

a measure organically connected to a
■ conceptual model

(1) Introduction



Indicators should be developed and managed so that they ...

- ... represent different aspects of the reality,*
- ... picture the reality in an interpretable way,*
- ... allow meaningful stories to be told and*
- ...support evaluations and decisions*

(1) Introduction

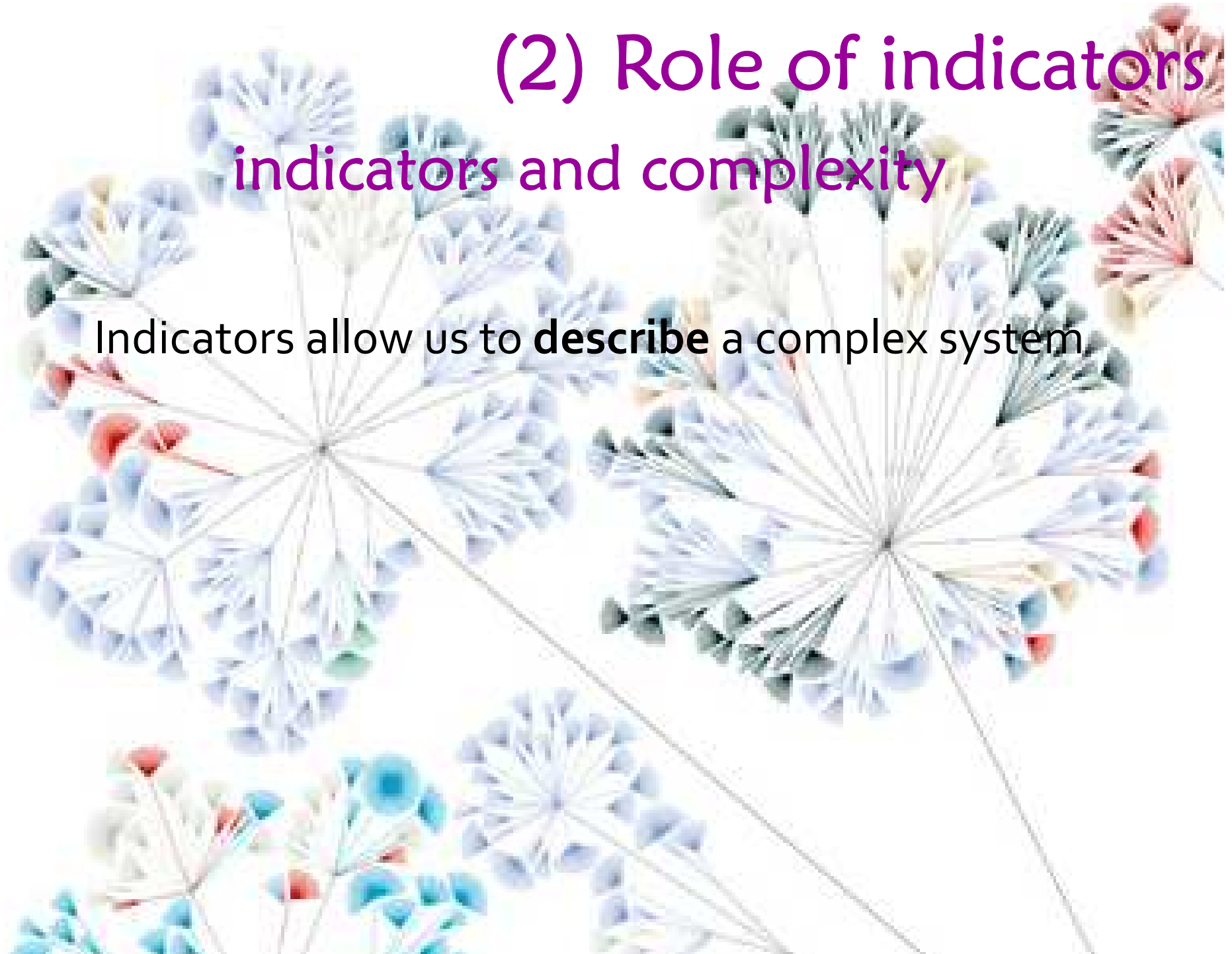




(2) Role of indicators

(2) Role of indicators indicators and complexity

Indicators allow us to **describe** a complex system





(2) Role of indicators indicators and complexity

Indicators allow us to **describe** a complex system

Indicators allow us to **understand** a complex system



(2) Role of indicators indicators and complexity

Indicators allow us to **describe** a complex system

Indicators allow us to **understand** a complex system

Indicators allow us to **control** a complex system



(2) Role of indicators indicators and complexity

Indicators allow us to **describe** a complex system

Indicators allow us to **understand** a complex system

Indicators allow us to **control** a complex system

*systemic view of
multi-indicator system*

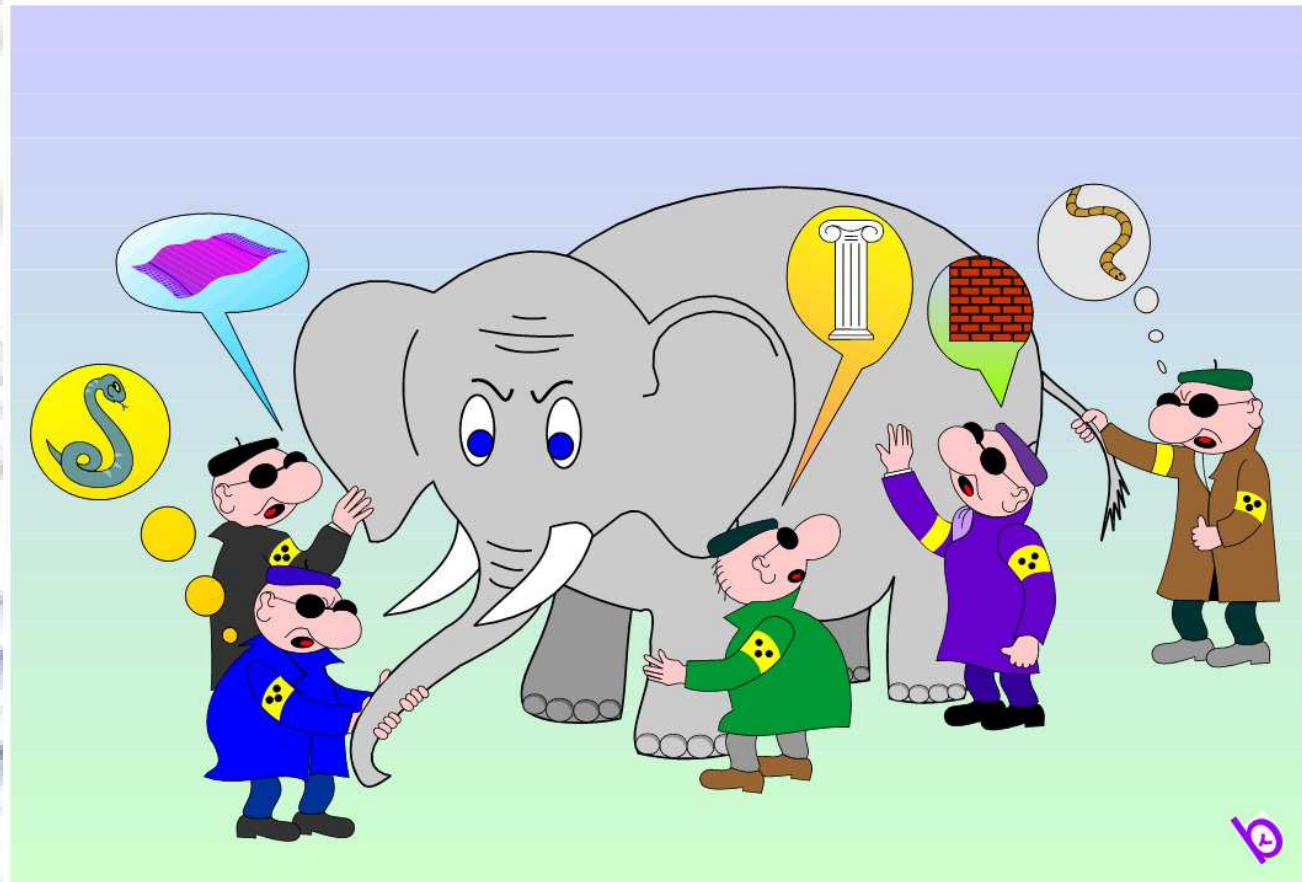
(2) Role of indicators

indicators and complexity

describing

understanding

controlling



multi-indicator system



(2) Role of indicators indicators and complexity

Consequences of complexity in constructing indicators

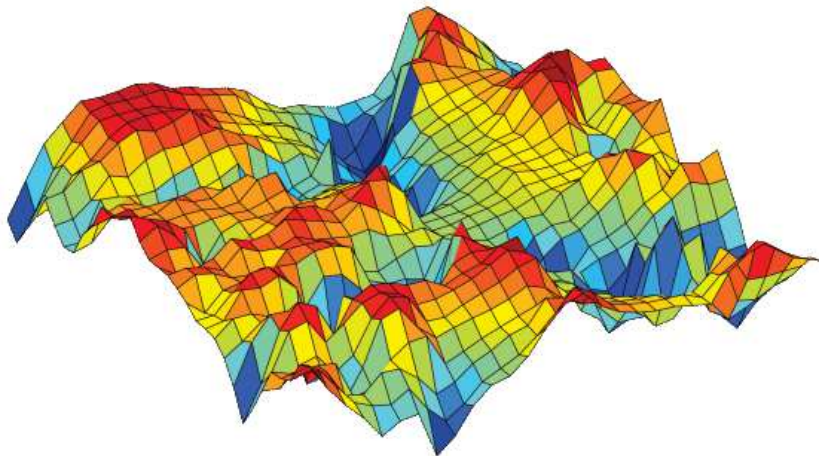
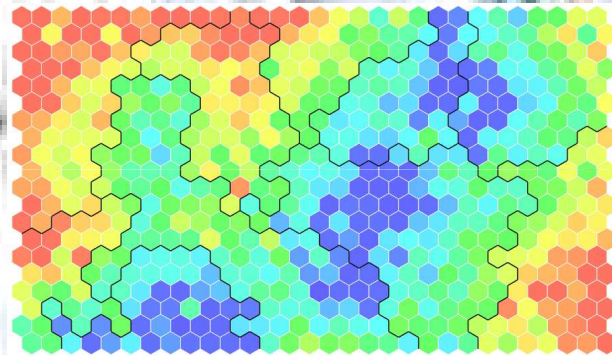
An indicator can be an **object**

able to preserve the complexity by stylizing it

(2) Role of indicators indicators and complexity

Consequences of complexity in constructing indicators

a map





(2) Role of indicators indicators and complexity

Consequences of complexity in constructing indicators

This has methodological consequences



*what we are going to construct should be an **authentic representation** of the reality*



(2) Role of indicators indicators and complexity

Consequences of complexity in constructing indicators

This has methodological consequences



not a compress / pointfold / pointform representation

but

(2) Role of indicators indicators and complexity

Consequences of complexity in constructing indicators

This has methodological consequences



not a compress / pointfold / pointform representation

but

*a representation preserving the systemic characteristic of the
phenomena*

*defined by **elements and their relationships***

(2) Role of indicators indicators and complexity

Consequences of complexity in constructing indicators

This has methodological consequences



from

points (numbers)

to

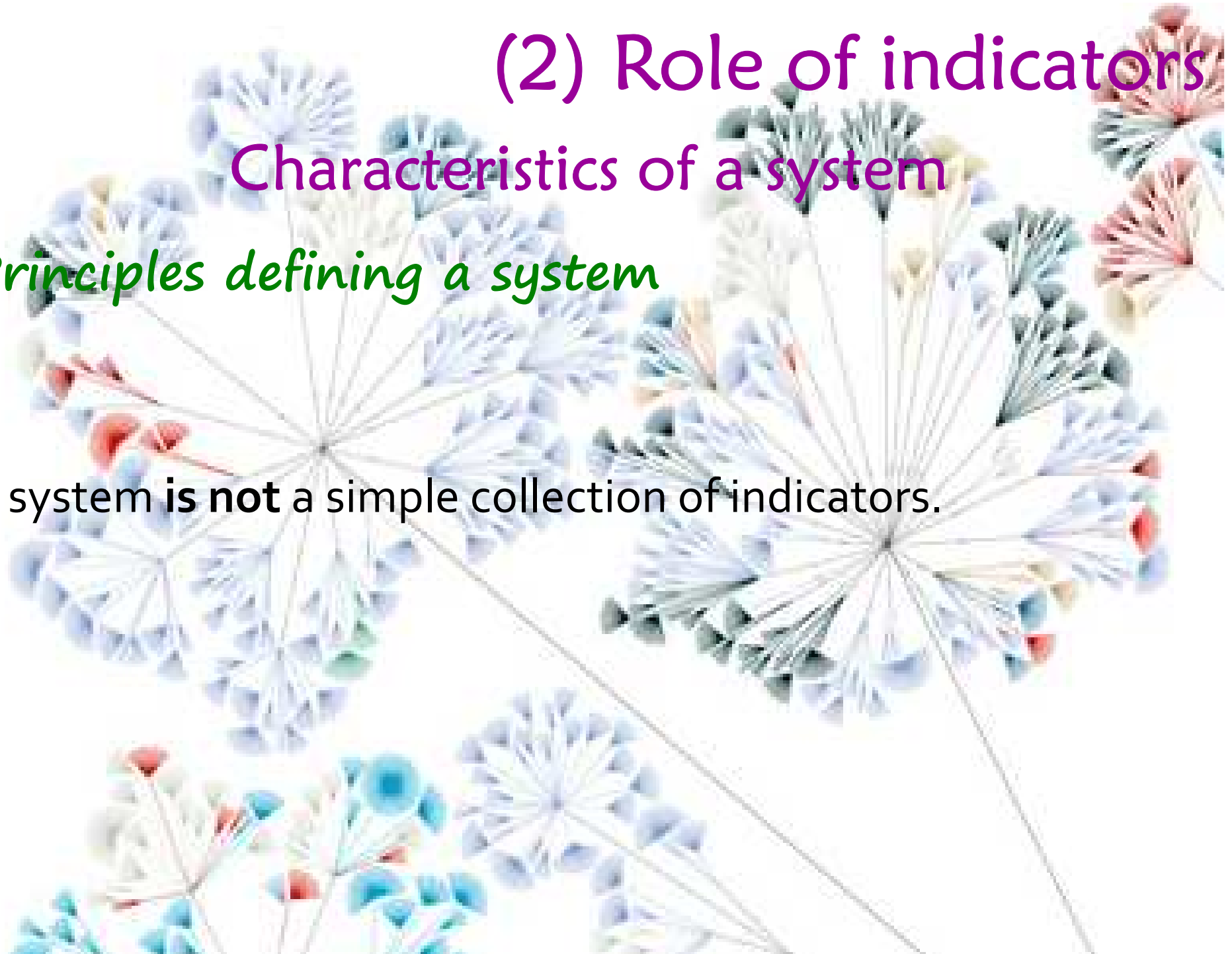
pattern (simplified shape and structure)

(2) Role of indicators

Characteristics of a system

Principles defining a system

A system **is not** a simple collection of indicators.



(2) Role of indicators

Characteristics of a system

Principles defining a system

A system **is** an interconnected set of elements, organized consistently with a perspective.

(2) Role of indicators

Characteristics of a system

Principles defining a system

integrity

A system is more than the sums of parts.

"It may exhibit adaptive, dynamic, goal-seeking, self-preserving, and sometimes evolutionary behaviour"

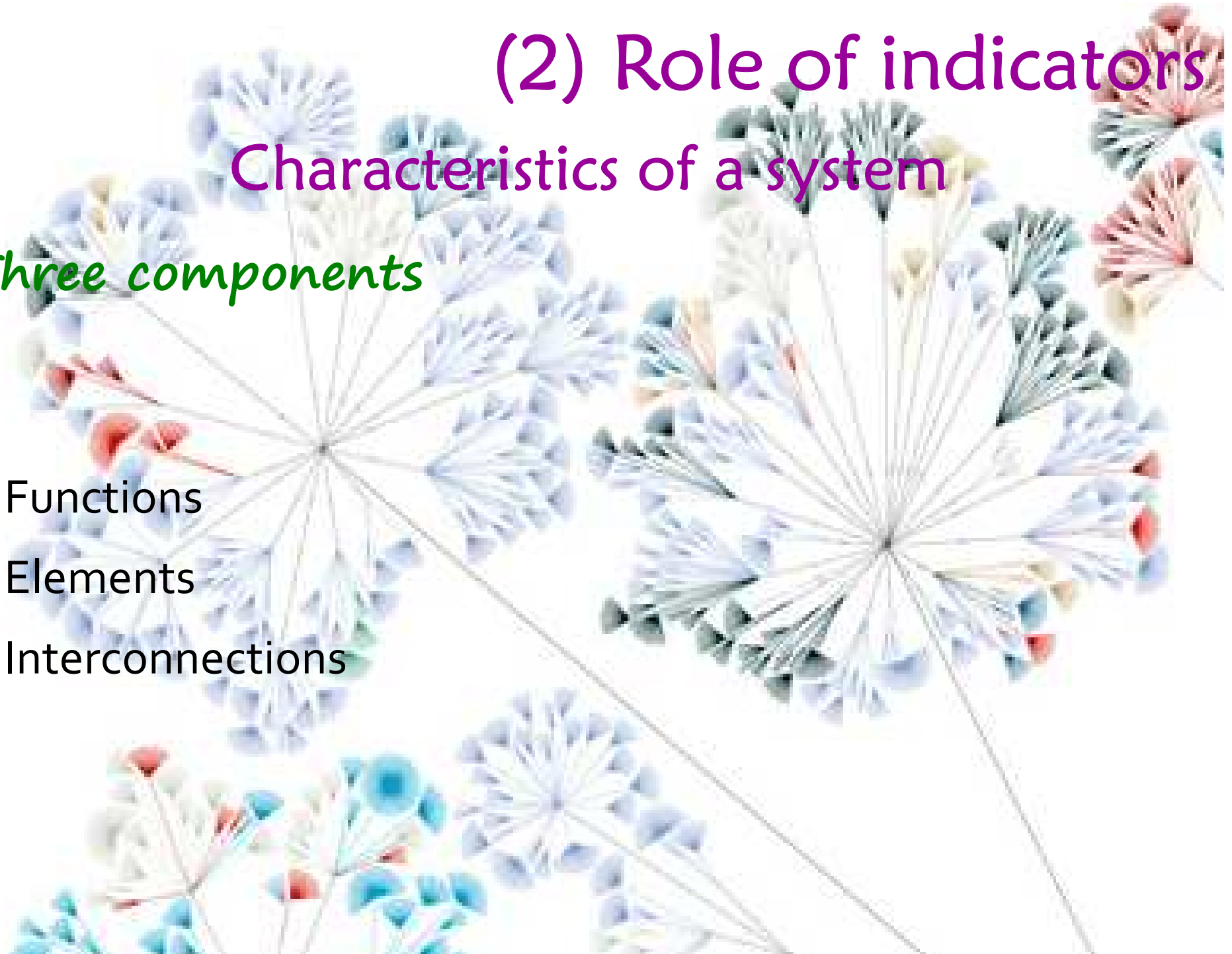
(Meadows, 2008)

(2) Role of indicators

Characteristics of a system

Three components

- Functions
- Elements
- Interconnections



(2) Role of indicators

Characteristics of a system

Three components

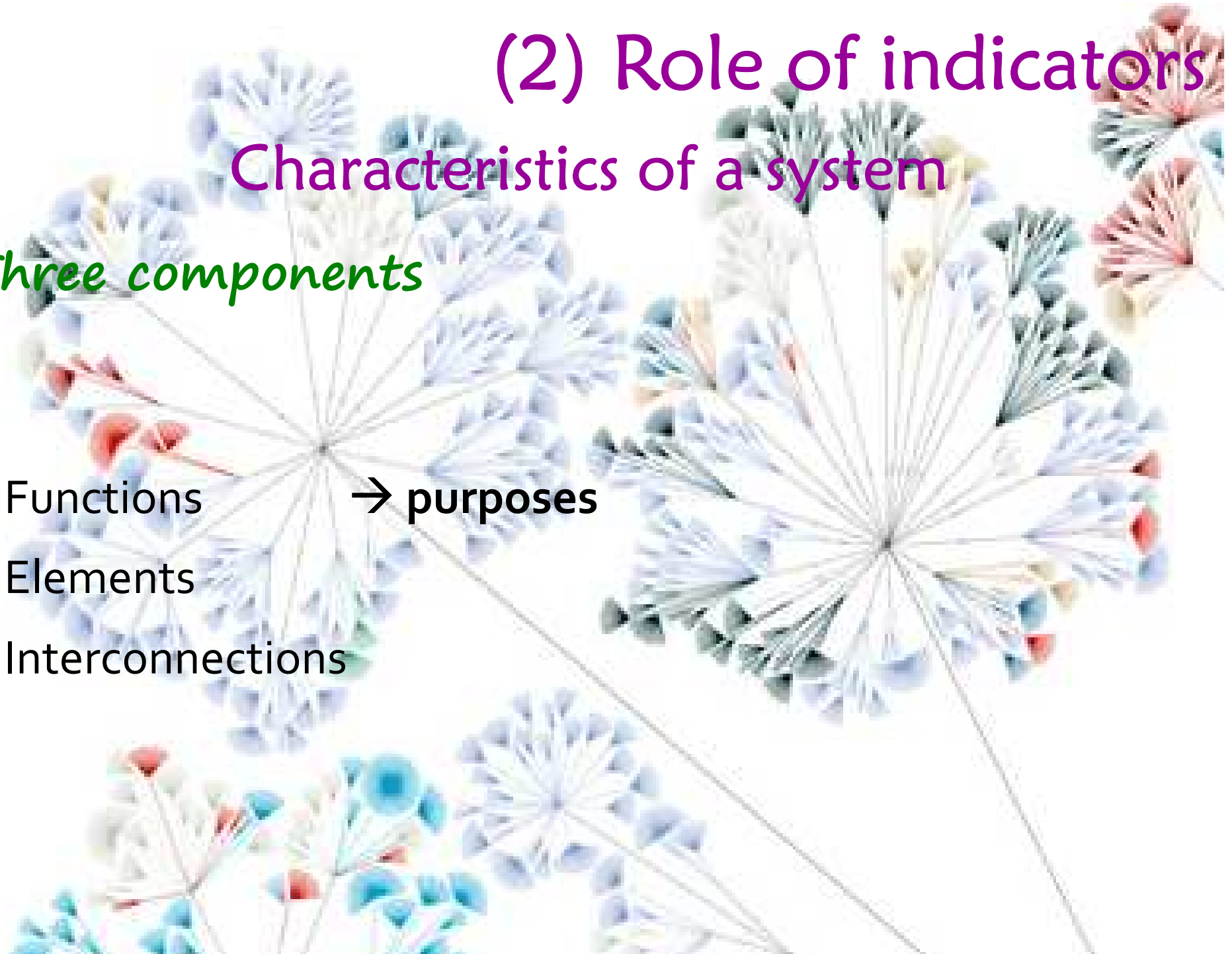
- Functions → the most crucial determinant of the system and its functioning
- Elements
- Interconnections

(2) Role of indicators

Characteristics of a system

Three components

- Functions → purposes
- Elements
- Interconnections



(2) Role of indicators

Characteristics of a system

Three components

- Functions → purposes
- Elements → indicators
- Interconnections

(2) Role of indicators

Characteristics of a system

Three components

- Functions → **purposes**
- Elements → **indicators**
- Interconnections → relationships holding elements together
flows of signals and information

(2) Role of indicators

Characteristics of a system

Three components

- Functions → purposes
- Elements → indicators
- Interconnections → relationships

The image features a collage of various flowers, including purple, blue, and red blooms, set against a white background. The text "(3) How to obtain indicators" is overlaid in a purple font, centered on the image.

(3) How to obtain indicators

(3) How to obtain indicators

Process allowing indicators to be developed



starts from a query, a need of knowledge

(3) How to obtain indicators

deterministic modeling

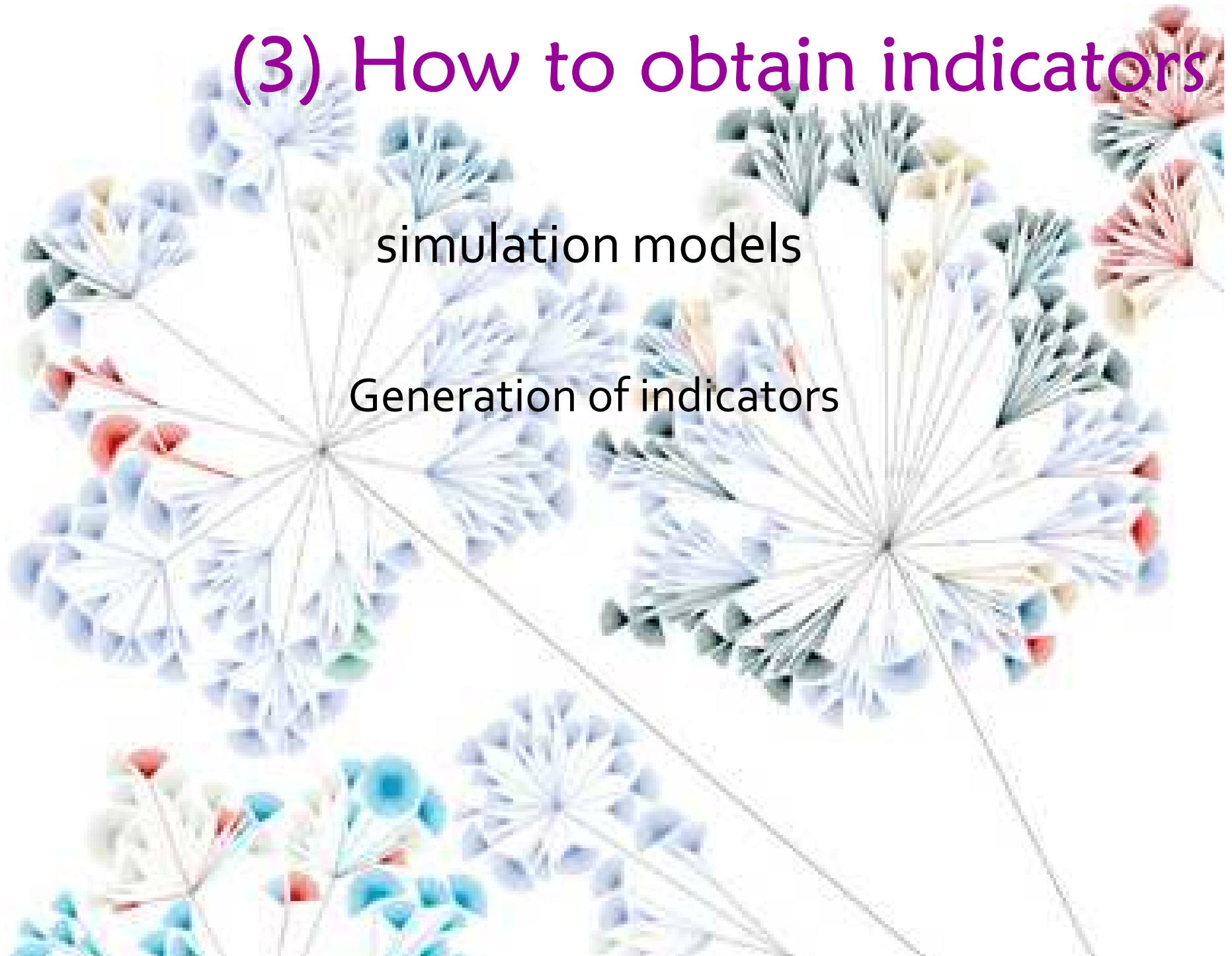
Sometimes deterministic modelling is possible, however data availability is difficult. Then indicator systems are useful.

Example: Newlin&Patil [1]

(3) How to obtain indicators

simulation models

Generation of indicators



(3) How to obtain indicators

Sometime a system can be described / understood and controls only by generating indicators



Project a system into a set of indicators

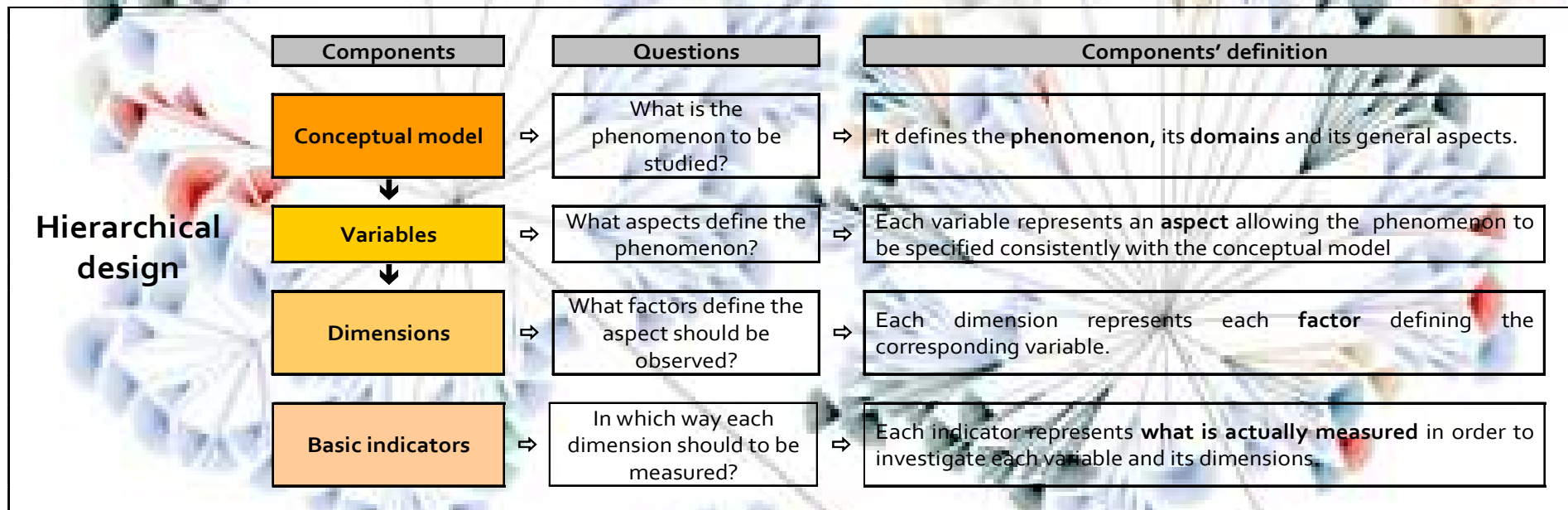
(3) How to obtain indicators

Process allowing indicators to be developed

the hierarchical design

which requires the definition of the different
subsequent components

(3) How to obtain indicators



(3) How to obtain indicators

Proper and accurate application of the hierarchical design



complex structure



each indicator measures and represents a distinct component in the description of the phenomenon

(3) How to obtain indicators

Proper and accurate application of the hierarchical design



complex structure



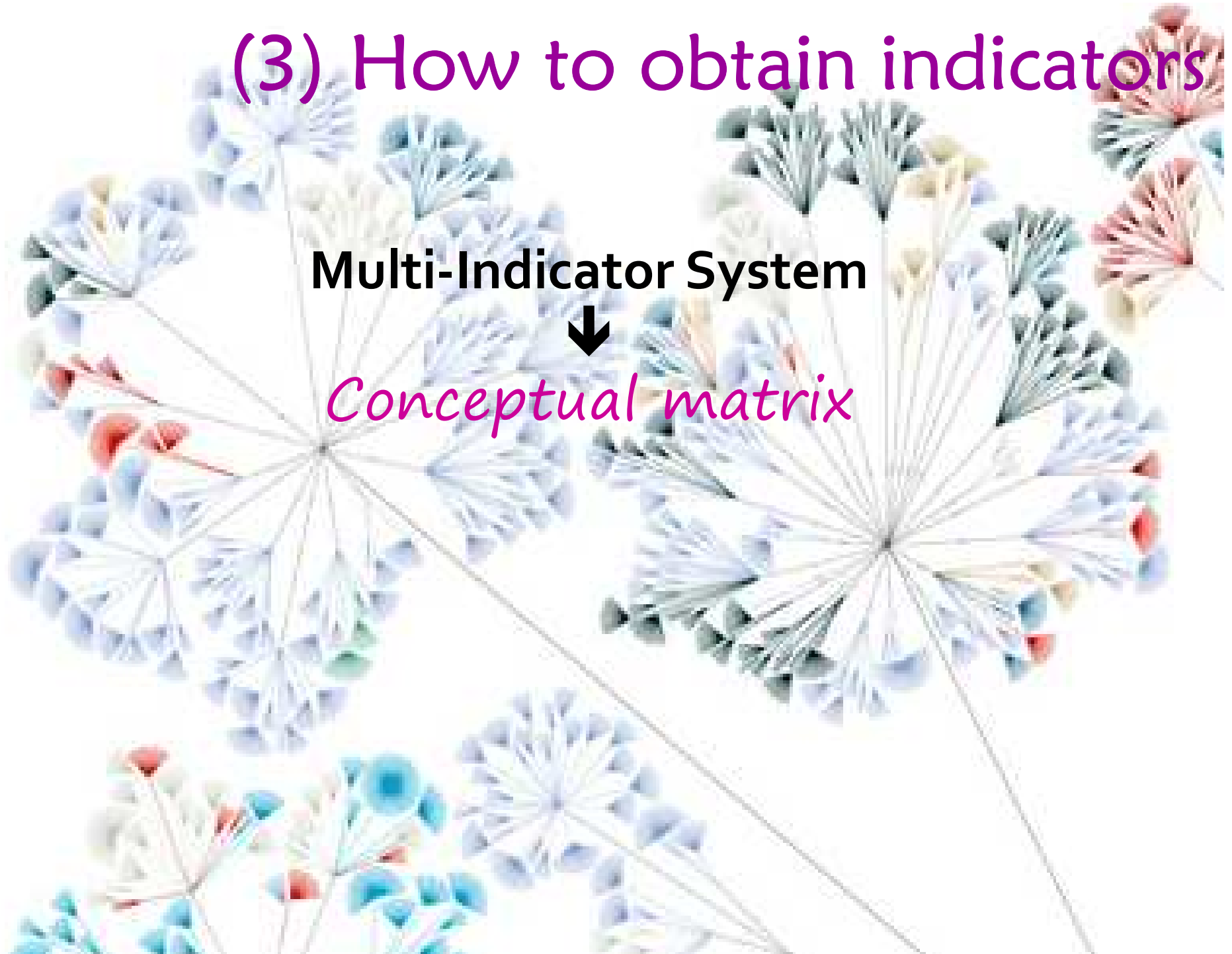
Multi-Indicator System

(3) How to obtain indicators

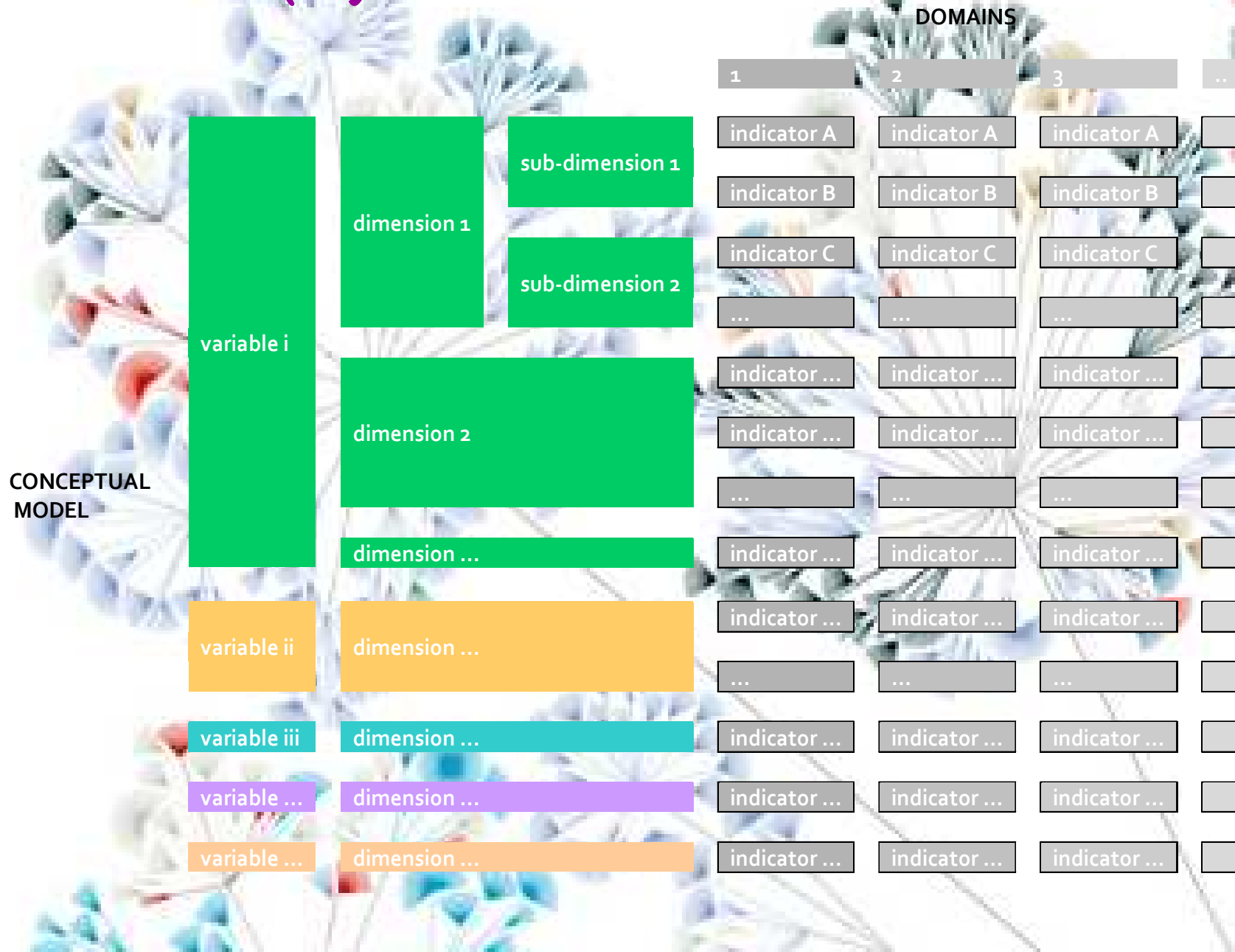
Multi-Indicator System



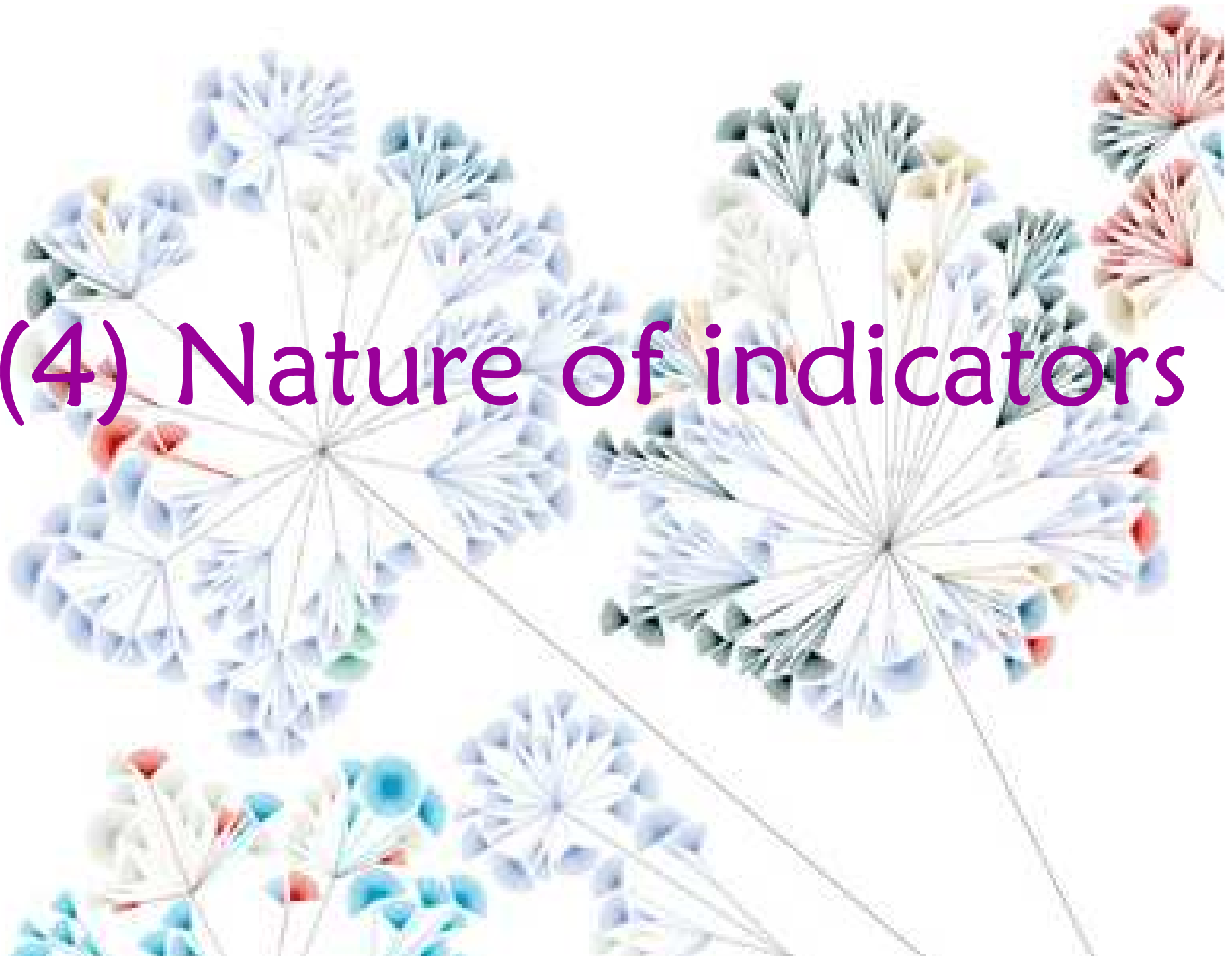
Conceptual matrix



(3) How to obtain indicators

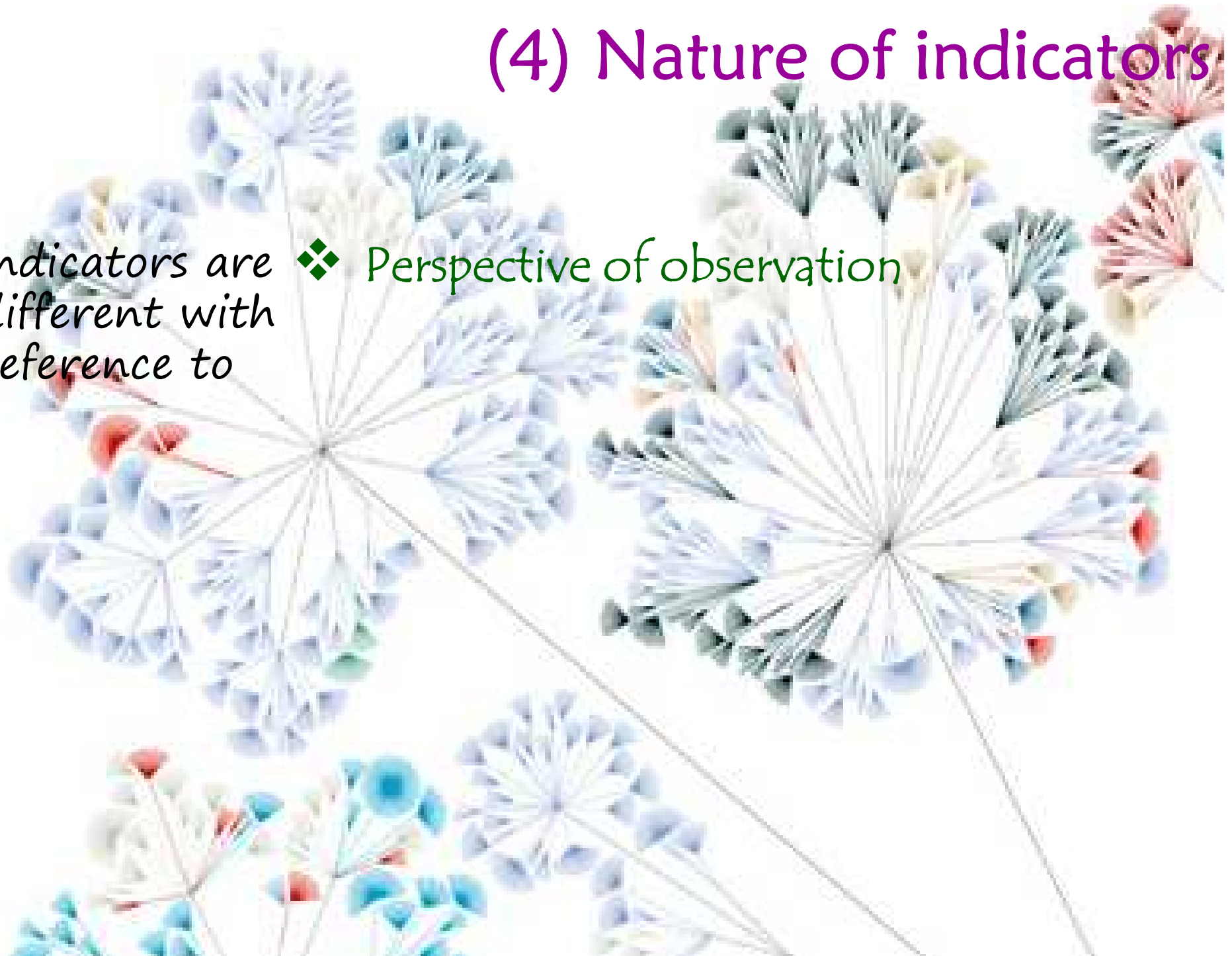


(4) Nature of indicators



(4) Nature of indicators

Indicators are different with reference to ❖ Perspective of observation



(4) Nature of indicators

Indicators are
different with
reference to

conglomerative \leftrightarrow deprivational

input \leftrightarrow outcome

positive \leftrightarrow negative

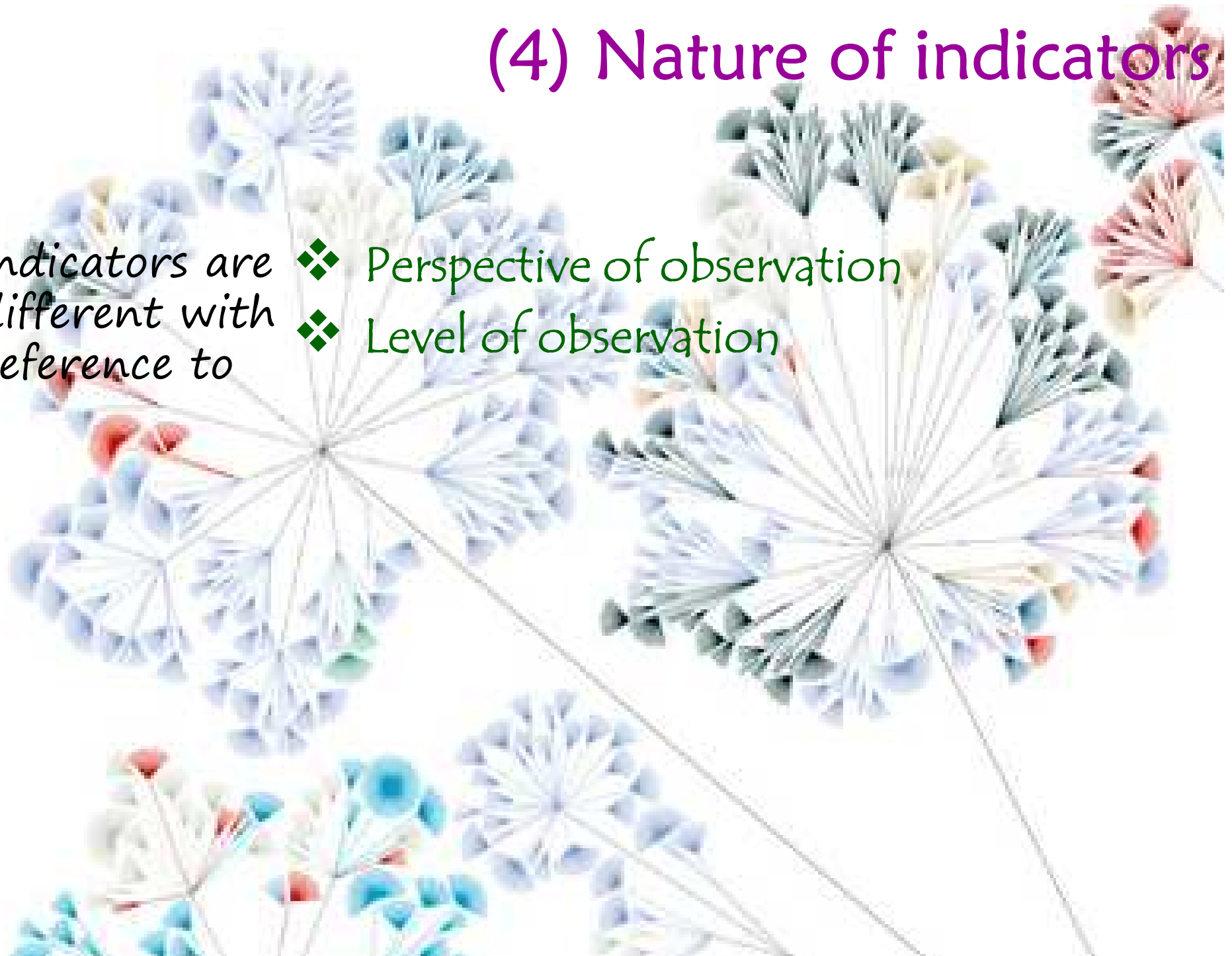
benefits \leftrightarrow costs

status \leftrightarrow trends

(4) Nature of indicators

Indicators are different with reference to

- ❖ Perspective of observation
- ❖ Level of observation

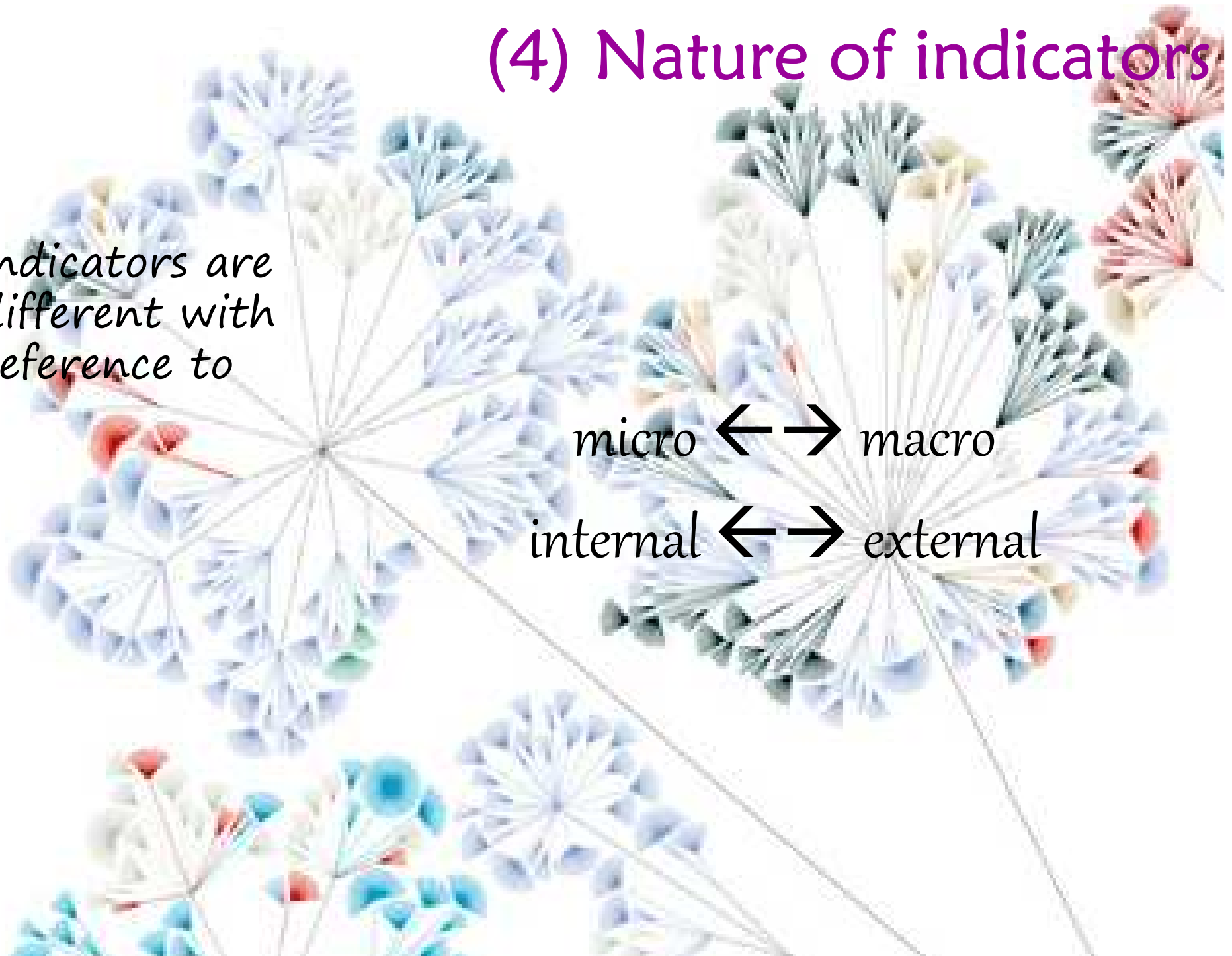


(4) Nature of indicators

Indicators are different with reference to

micro \leftrightarrow macro

internal \leftrightarrow external



(4) Nature of indicators

Indicators are different with reference to

- ❖ Perspective of observation
- ❖ Level of observation
- ❖ Nature of the observed characteristics

(4) Nature of indicators

Indicators are different with reference to

objective \leftrightarrow subjective

quantitative \leftrightarrow qualitative

(4) Nature of indicators

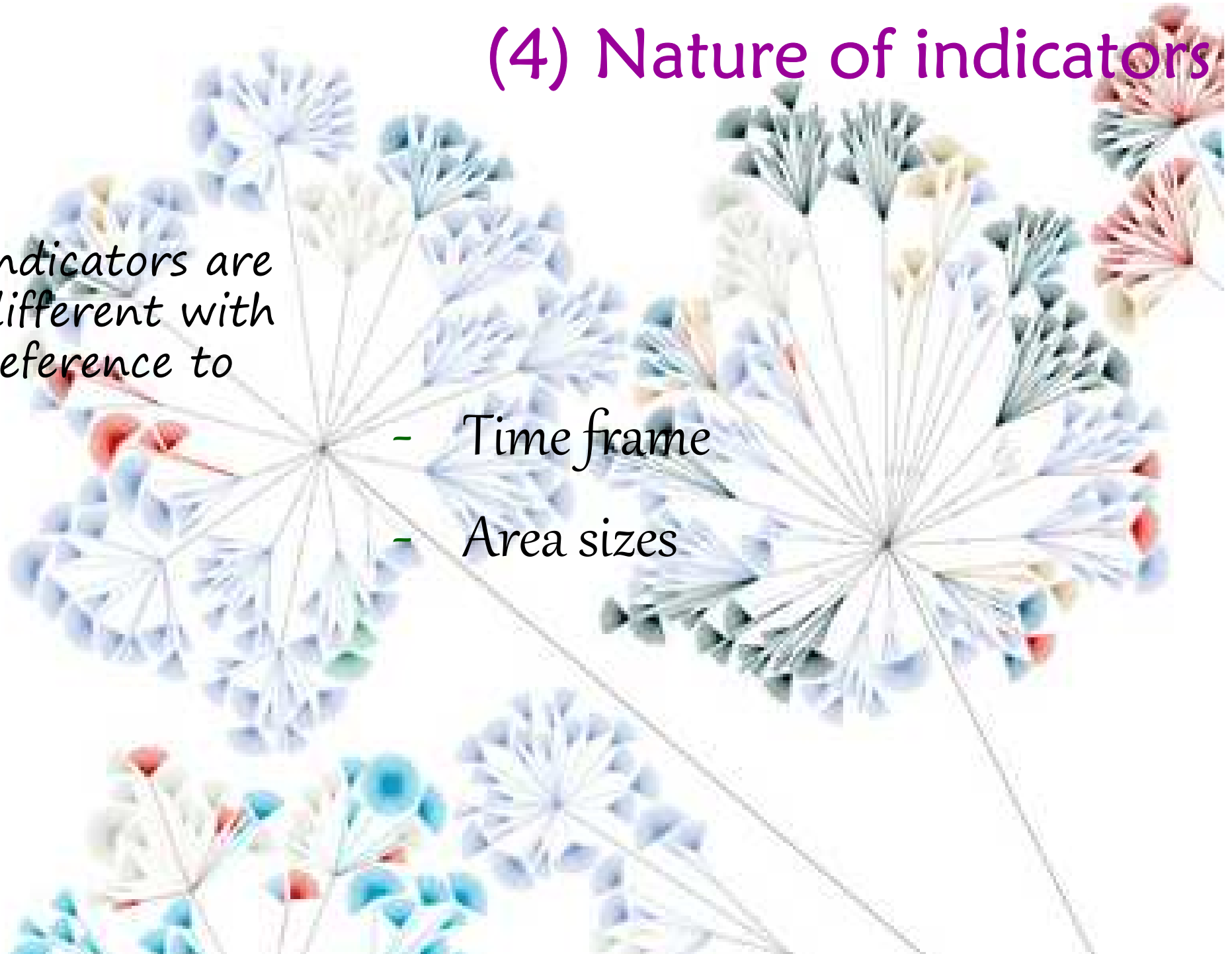
Indicators are different with reference to

- ❖ Perspective of observation
- ❖ Level of observation
- ❖ Nature of the observed characteristics
- ❖ Level of dis/aggregation

(4) Nature of indicators

Indicators are different with reference to

- Time frame
- Area sizes



(4) Nature of indicators

Indicators are different with reference to

- ❖ Perspective of observation
- ❖ Level of observation
- ❖ Nature of the observed characteristics
- ❖ Level of dis/aggregation
- ❖ Criteria

(4) Nature of indicators

Indicators are different with reference to

- goals
- identifying benchmarks
- reference standards
- ...

(4) Nature of indicators

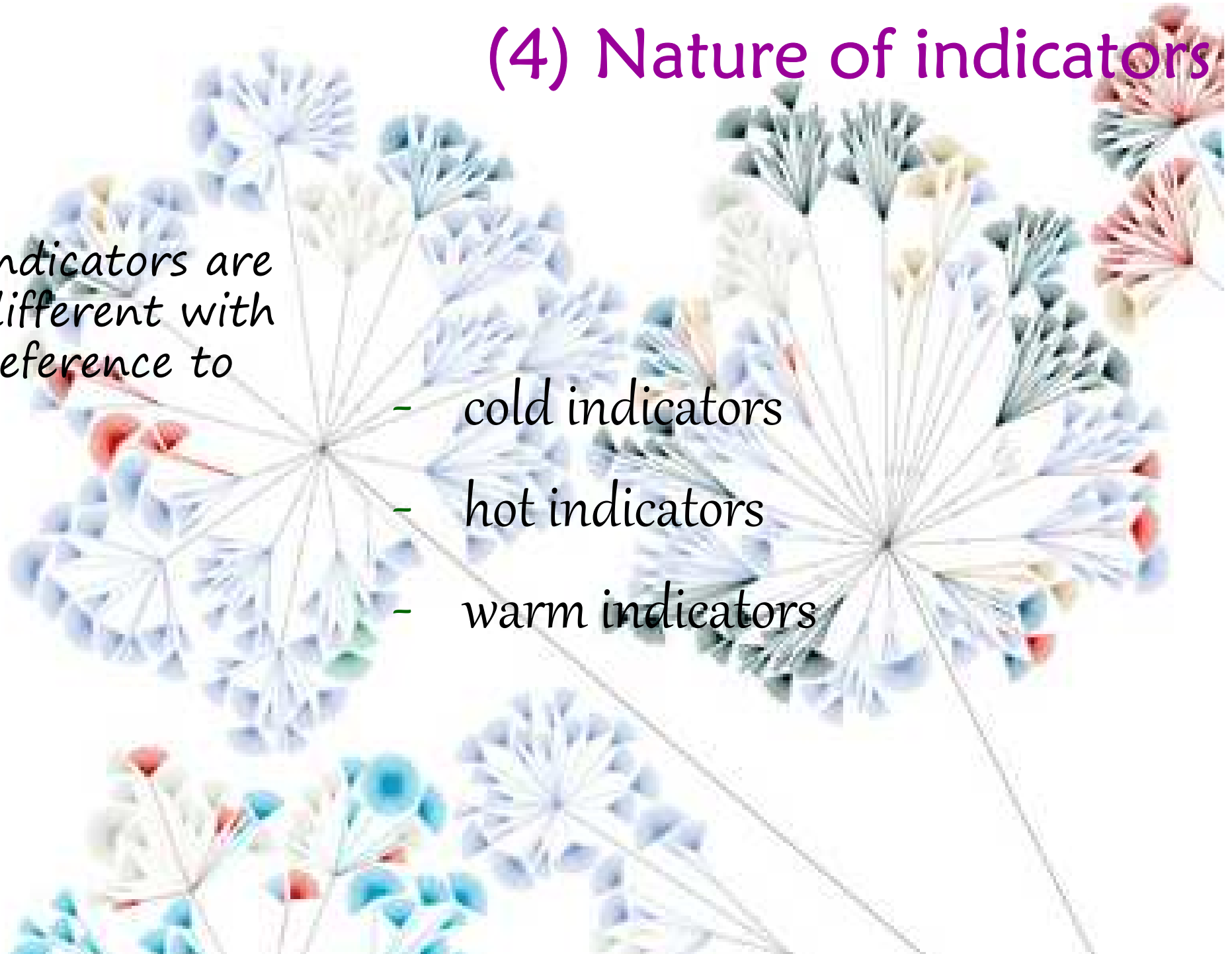
Indicators are different with reference to

- ❖ Perspective of observation
- ❖ Level of observation
- ❖ Nature of the observed characteristics
- ❖ Level of dis/aggregation
- ❖ Criteria
- ❖ Levels of complication

(4) Nature of indicators

Indicators are different with reference to

- cold indicators
- hot indicators
- warm indicators



(4) Nature of indicators

Indicators are different with reference to

- ❖ Perspective of observation
- ❖ Level of observation
- ❖ Nature of the observed characteristics
- ❖ Level of dis/aggregation
- ❖ Criteria
- ❖ Levels of complication
- ❖ Purposes

(4) Nature of indicators

Indicators are different with reference to

- descriptive
- explicative
- predictive
- normative
- problem-oriented
- evaluating

(4) Nature of indicators

Indicators are different with reference to

- ❖ Perspective of observation
- ❖ Level of observation
- ❖ Nature of the observed characteristics
- ❖ Level of dis/aggregation
- ❖ Criteria
- ❖ Levels of complication
- ❖ Purposes
- ❖ Governance context

(4) Nature of indicators

Indicators are different with reference to

- public debate
- policy governance
- administrative guidance

(4) Nature of indicators

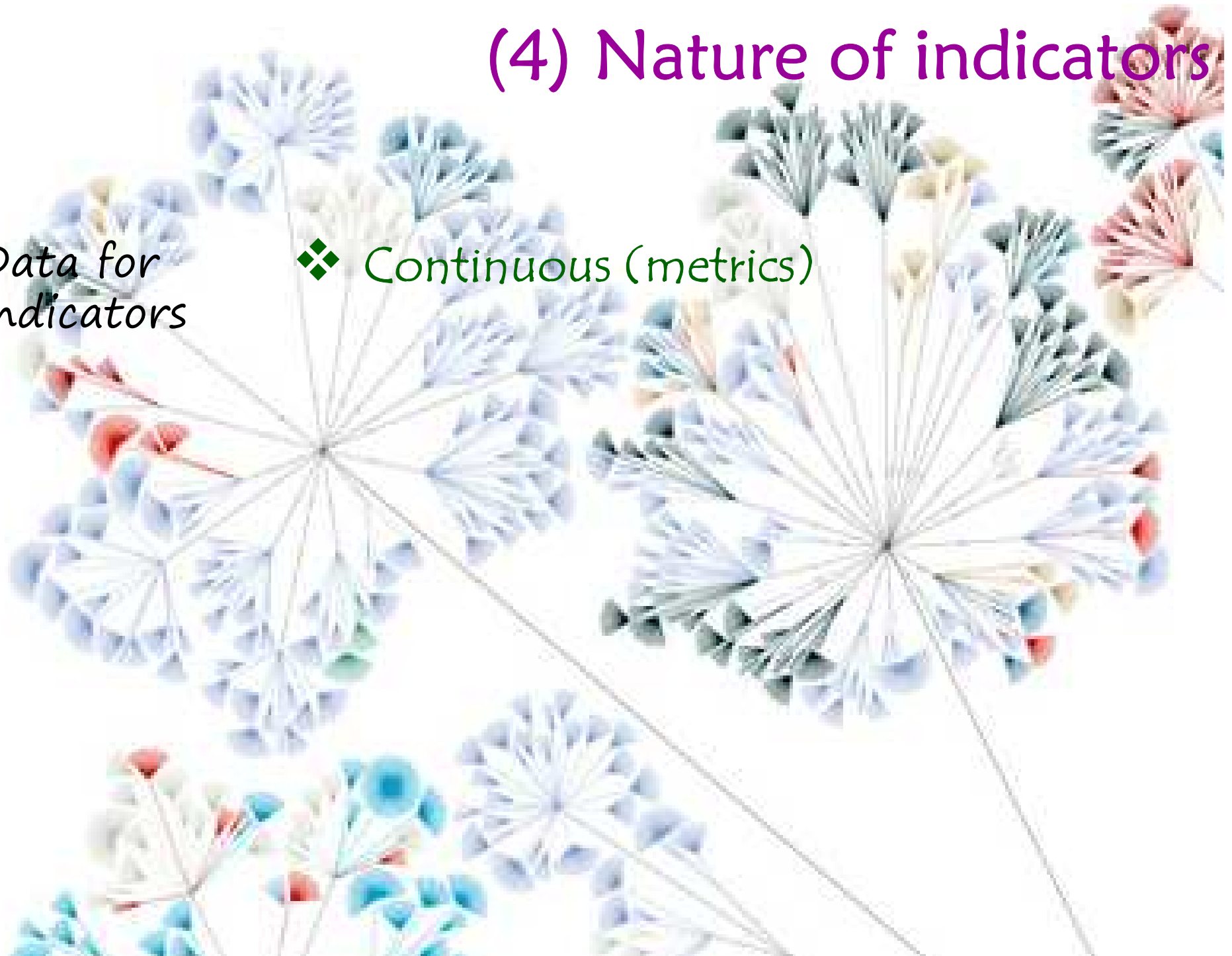
Indicators are different with reference to

- ❖ Perspective of observation
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- ❖ Nature of the observed characteristics
- ❖ Level of dis/aggregation
- ❖ Criteria
- ❖ Levels of complication
- ❖ Purposes
- ❖ Governance context
- ❖

(4) Nature of indicators

*Data for
indicators*

❖ Continuous (metrics)



(4) Nature of indicators

Data for
indicators

- ❖ Continuous (metrics)
- ❖ Discrete (counts)

Operations of a linear space can be performed – if meaningful
(weightings, weighted sum as utility function)

(4) Nature of indicators

Data for indicators

- ❖ Continuous (metrics)
- ❖ Discrete (counts)
- ❖ Ordinal (ratings, ranks)

At least comparisons are possible

(4) Nature of indicators

Data for indicators

- ❖ Continuous (metrics)
- ❖ Discrete (counts)
- ❖ Ordinal (ratings, ranks)
- ❖ Nominal (categories)

Descriptive, but restricted applicability. For example: stratification



(5) Indicators and ranking

(5) Indicators and ranking

Indicators for evaluation
and orientation

- ❖ How many
 - too few indicators
 - too many indicators → information noise
- ❖ How do they relate to each other → weakness of traditional approaches

Cluster analysis, PCA, dimension reduction techniques.



networks

(5) Indicators and ranking

Preconditions for ranking

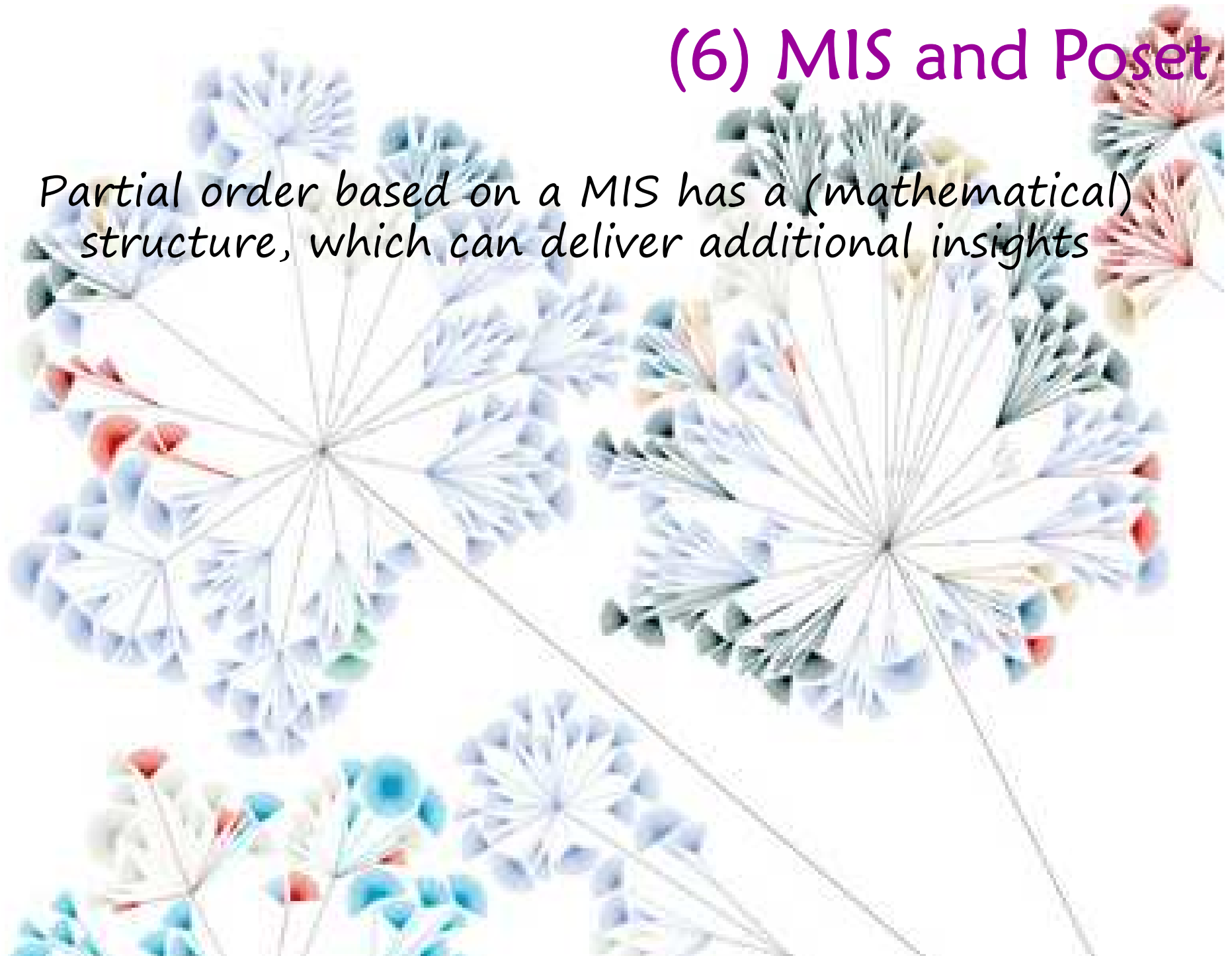
- A aim for ranking must be defined
- A suitable set of indicators must be found
- The set of indicators must have the following properties
 1. Orientability: accordance with the ranking aim
 2. Ordinability: any indicator induces at least a weak order



(6) MIS and Poset

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Partial order based on a MIS has a (mathematical) structure, which can deliver additional insights



(6) MIS and Poset

Partial order based on a MIS has a (mathematical) structure, which can deliver additional insights

Mathematical structure of PO allows data structure to be explored: separateness / isolation / grids / sensitivity / antichains / CAM-graph (role of fine-tuning) → Marco's approaches

(6) MIS and Poset

Partial order based on a MIS has a (mathematical) structure, which can deliver additional insights

Mathematical structure of PO allows data structure to be explored: separateness / isolation / grids / sensitivity / antichains / CAM-graph (role of fine-tuning) → Marco's approaches

Question: how many indicators are needed to obtain a reliable partial order

(6) MIS and Poset

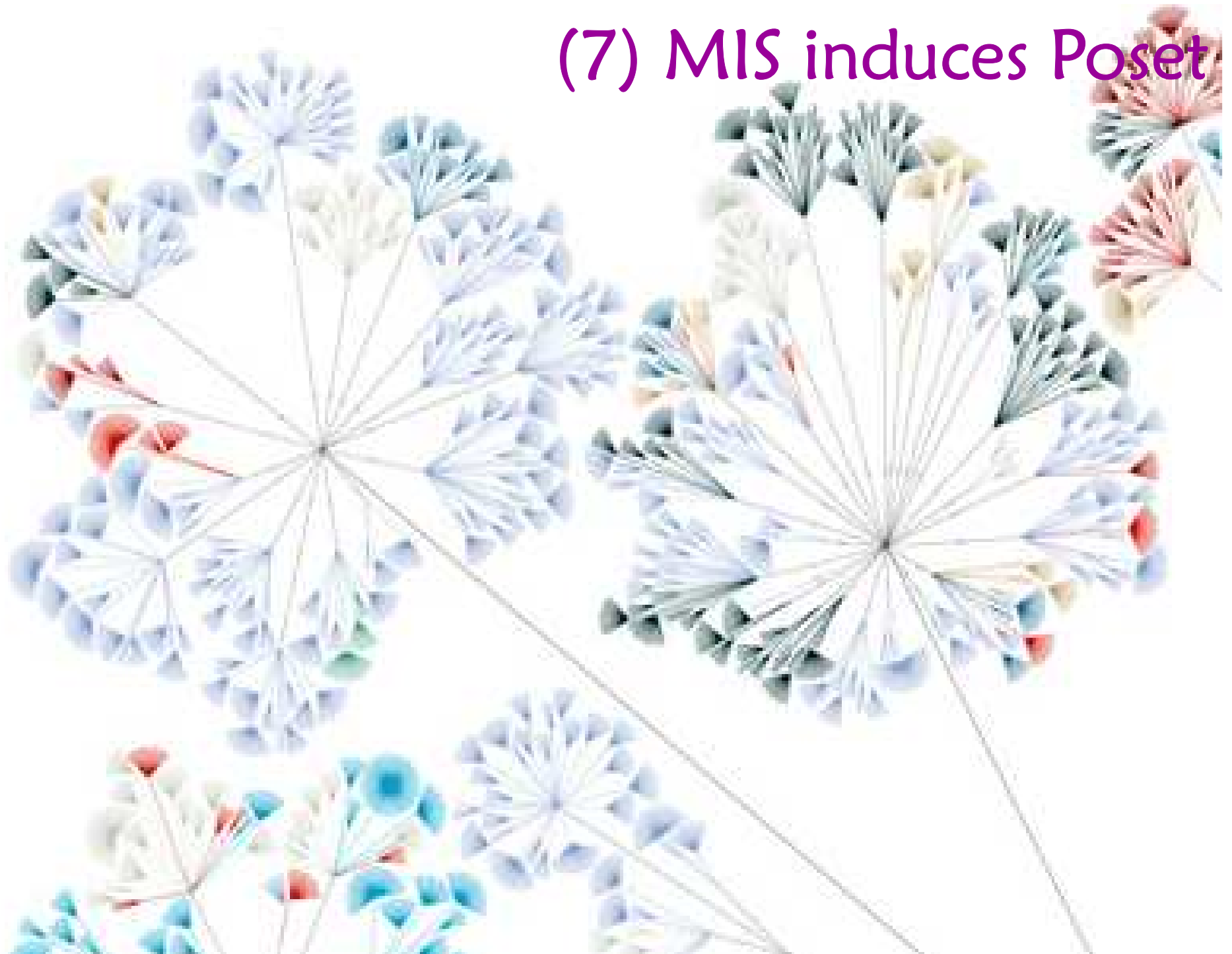
If indicators are discrete and ordinal with range $[0, m(j)]$:

Two variants: posets based on

a) $\Pi[0, m(j)]$, Fattore-approach \rightarrow grid

b) $\Pi[0, m(j)]$ realized by objects \rightarrow no grid, but graph-theoretical structure

(7) MIS induces Poset



(7) MIS induces Poset

Hypothesis

- Whether or not a MIS is more than the sum of its constituents depends on the representation in the ranking
- Local representations (parallel coordinates)
(see e.g. Rocco&Ramirez-Marquez [4]), amoeba (spider) diagrams, glyphs,...)
- Global representation by directed graphs (Hasse diagrams)

(7) MIS induces Poset

Global representation of orders by a directed graph

- Let q_1, q_2, \dots, q_m be a MIS and let q_j have at least an ordinal scaling level, then:
- A profile „ v “ is an ordered set of values v_j of q_j ($j=1, \dots, m$):
(v_1, v_2, \dots, v_m)
- Two profiles labelled by k and k' fulfill a partial order relation if $v_{jk} \leq v_{jk'}$ for all j .
- By applying the cover relation a directed graph can be obtained, called a **Hasse diagram** (HD)

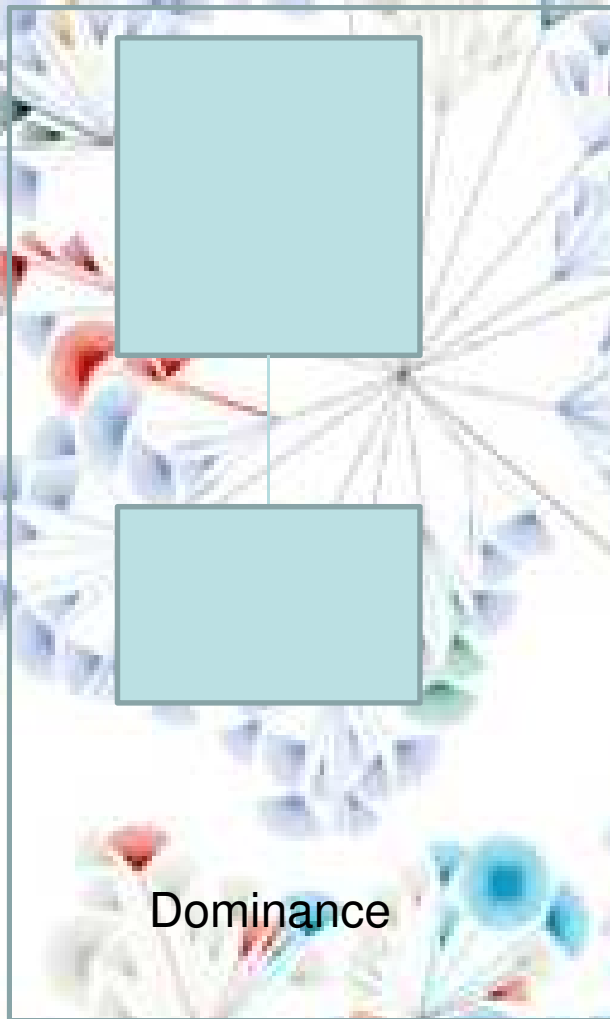
(7) MIS induces Poset

Global presentation by a HD

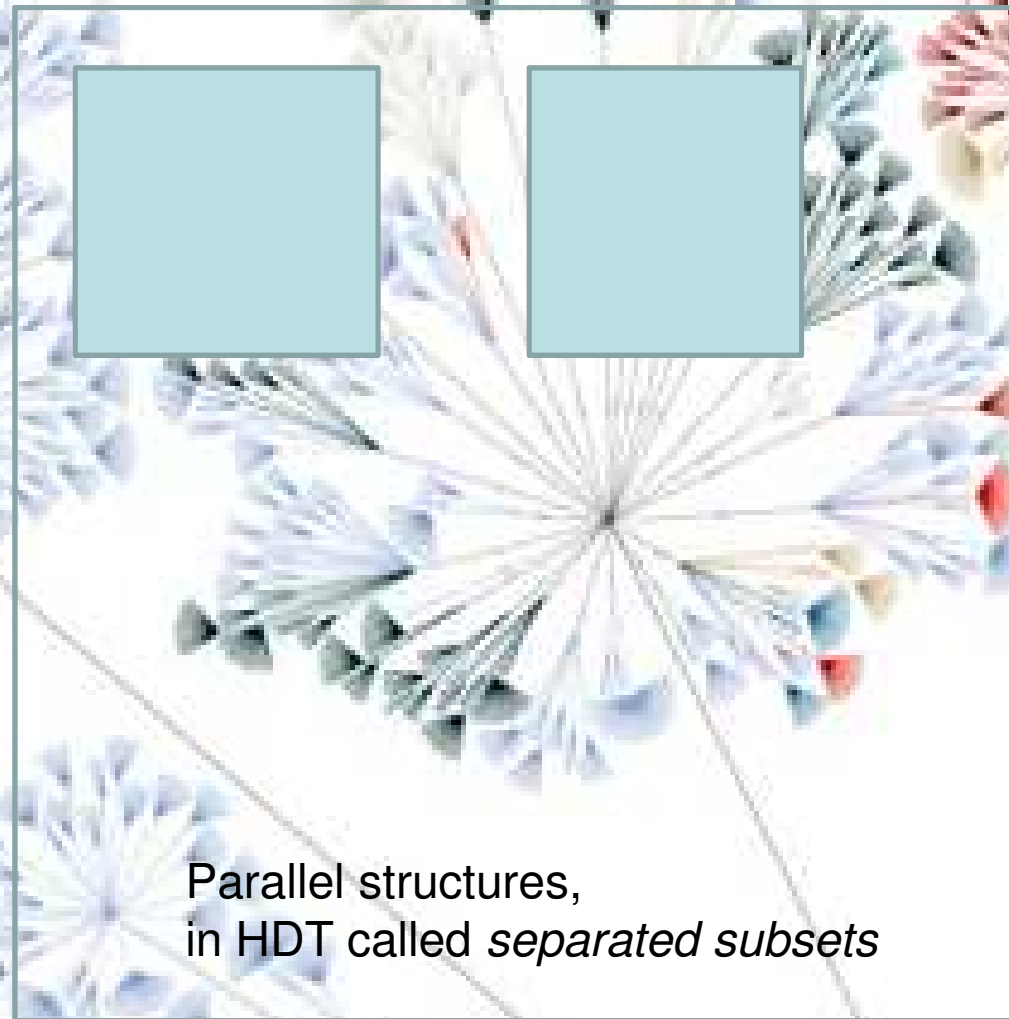
- The Hasse diagram is a global presentation of a partial order, because it shows **all** possible relations among the profiles.
- **Only the global representation** by directed graphs illustrates the surplus in a MIS

(7) MIS induces Poset

Structures of object-related Hasse diagrams



Dominance



Parallel structures,
in HDT called *separated subsets*

Both, cannot be seen by considering any single indicator but is a property of the whole MIS

(7) MIS induces Poset

Indicators and their information content

- If a ranking is intended, then the set of indicators bears the relevant information
- Amoeba diagrams (and other techniques) yield just what the numerical values of each single indicator tells us.
- The „object-related“ Hasse diagram has a structure which tells us more than any single indicator
- The structure in turn is the result of incomparabilities

(7) MIS induces Poset

Information and incomparabilities, U

- As known, if indicators are selected for a ranking, then they must be orientable and this characteristic induces at least a weak order
- There is a trade-off: one and only one indicator induces at least a weak order:
 $U = 0$, perfect decision, because any object is comparable to any other, no structure.

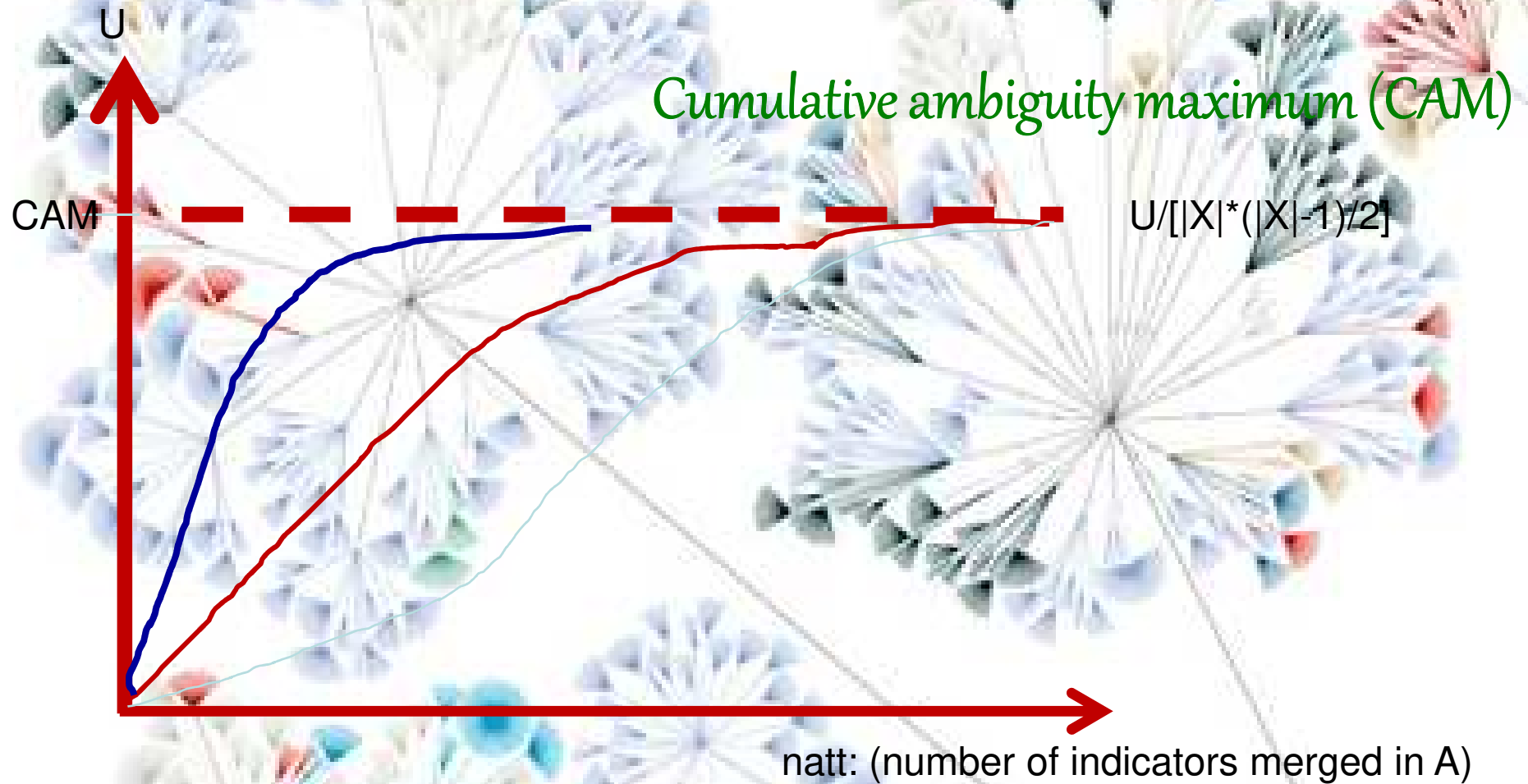
(7) MIS induces Poset

Information and incomparabilities, U

- However, with many indicators a decision is difficult
- When the number of indicators is large enough ($U \rightarrow |X| * (|X|-1)/2$)
no structure, no decision (at least within the level of the Hasse diagram itself)

(7) MIS induces Poset

Information and incomparabilities, U



The graph $U=f(\text{natt})$ is always weakly monotonous increasing with natt

Discussion

- Indicator set: a value for its own right
- Typical questions: Latent indicators; Synthetic indicators
- Indicator set for Ranking: MIS
- Representing MIS through an object-related Hasse diagram has a surplus
- Not mentioned: the possibility of poset theory to construct latent indicators


Discussion



- Structure of value-related Hasse diagrams, governed by generating functions
- Structure of object-related Hasse diagrams: a consequence of incomparability
- In comparison to value-related Hasse diagram, the effect of incomparability can be more pronounced: separability

Discussion

- There is still no validation for partitioning the indicator set due to the Am-function.
- Heuristically, there are good arguments that those indicators merged into the set A which are beyond p% may be considered as finetuning. However, can these indicators really be considered as information noise?

A vibrant, colorful floral arrangement featuring various flowers in shades of purple, blue, yellow, and red, set against a white background. The flowers are arranged in a circular pattern, with some larger, more prominent blooms in the center and smaller ones towards the edges. The overall composition is bright and cheerful.

Thank you for your attention!