Laser Physics, Vol. 15, No. 11, 2005, pp. 1–6.

On the Stability of Optical Lattices[1]

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Received May 17, 2005

Abstract—In this article, we present an analysis of the stability of optical lattices. Starting with the study of an unstable optical lattice, we establish a necessary and sufficient condition for intrinsic phase stability and discuss two practical solutions to fulfill this condition, namely, minimal and folded optical lattices. We then present a particular example of two-dimensional folded optical lattice, which has the advantages of being symmetric, possessing power recycling, and having a convenient geometry. We used this lattice for laser collimation of a continuous cesium beam in a fountain geometry.

1. INTRODUCTION

It is well known that the energy levels of an atom interacting with an electromagnetic field undergo an a.c. Stark or light shift [1, 2] proportional to the local light intensity. In a standing wave, the ground state light shift gives rise to a periodic potential, called an optical lattice, which can be used to trap the atoms [3–5]. The first experiments demonstrating the trapping of atoms in the potential wells of an optical lattice and their localization therein were performed by the groups of Grynberg [6] and Jessen [7]. Since then, studies of laser cooling, quantum state preparation, and Bose–Einstein condensates in optical lattices have intensified.

In this article, we present a general analysis of the stability of optical lattices. After a short presentation of the instability problem, we establish a necessary and sufficient condition for intrinsic phase stability. Then we discuss two practical solutions to guarantee intrinsic phase stability: the first of these, minimal optical lattices, is the solution proposed by Grynberg *et al.* in 1993 [8], while the second, folded optical lattices, was suggested by Rauschenbeutel *et al.* a few years later [9]. Finally, we discuss both approaches and describe a laser cooling experiment in which we have used a two-dimensional (2D) folded optical lattice for the collimation of a continuous atomic beam.

In the following, we consider an optical lattice resulting from the superposition of l laser beams. The total electric field is given by $\mathbf{E}_L(\mathbf{r}, t) = \mathrm{Re}[\mathbf{E}_L(\mathbf{r})\exp(-i\omega_L t)]$, where

$$\mathbf{E}_L(\mathbf{r}) = \sum_{j=1}^{l} E_j \boldsymbol{\varepsilon}_j \exp[i(\mathbf{k}_j \cdot \mathbf{r} + \phi_j)], \quad (1)$$

ϕ_j being the phase of the jth laser beam. As explained in [3] and [10], the optical shift operator for atoms in the ground state is given by

$$\hat{U}(\mathbf{r}) = -\mathbf{E}_L^*(\mathbf{r}) \cdot \hat{\alpha} \cdot \mathbf{E}_L(\mathbf{r}), \quad (2)$$

where $\hat{\alpha}$ is the atomic polarisability tensor operator given by $\hat{\alpha} = -\sum_e \hat{\mathbf{d}}_{ge}\hat{\mathbf{d}}_{eg}/\hbar\Delta_{ge}$, where Δ_{ge} is the laser detuning with respect to the atomic transition $|g\rangle \longrightarrow |e\rangle$ and $\hat{\mathbf{d}}_{eg}$ is the electric dipole operator between these levels. By inserting Eq. (1) into Eq. (2), we get

$$\hat{U}(\mathbf{r}) = -\sum_{i,j} (\boldsymbol{\varepsilon}_i^* \cdot \hat{\alpha} \cdot \boldsymbol{\varepsilon}_j) E_i^* E_j \times \exp[i(\phi_j - \phi_i)]\exp[i(\mathbf{k}_j - \mathbf{k}_i)\cdot \mathbf{r}], \quad (3)$$

which is our starting point for the stability analysis.

Although in the following stability analysis we concentrate on the optical shift operator (2), all other field-dependent operators of the problem (e.g., the optical pumping rate operator) are also determined by quadratic combinations of type (3). Therefore, the stability conditions will also be the same for these operators.

2. PROBLEM OF STABILITY

From Eq. (3), it is clear that the relative phases $\phi_j - \phi_i$ play a critical role. Any variation in one of these phases arising, for example, from the vibration of a mirror may manifest itself as a dramatic change in the optical potential.

To illustrate this problem, let us consider the 2D optical lattice shown in Fig. 1, where four laser beams intersect in a common plane, all linearly polarized within that plane.

[1] This article was submitted by the authors in English.

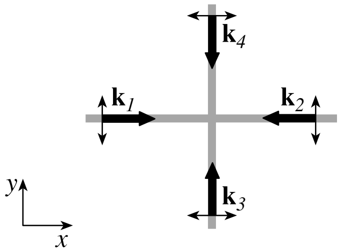

Fig. 1. Example of an unstable optical lattice of dimension 2.

The total electric field is given by the superposition of the electric fields of the four laser beams (with identical amplitude E):

$$\mathbf{E}_L(x, y) = E[\boldsymbol{\varepsilon}_y \exp(ikx + i\phi) + \boldsymbol{\varepsilon}_y \exp(-ikx) + \boldsymbol{\varepsilon}_x \exp(iky) + \boldsymbol{\varepsilon}_x \exp(-iky)], \tag{4}$$

where we have introduced the variable ϕ to represent a change in the phase of the first laser beam. Grouping terms with identical polarization, we obtain

$$\mathbf{E}_L(x, y) = 2E[\boldsymbol{\varepsilon}_x \cos(ky) + \boldsymbol{\varepsilon}_y \exp(i\phi/2)\cos(kx + \phi/2)]. \tag{5}$$

This expression shows that the total polarization is critically dependent on the angle ϕ. Indeed, if we calculate the intensity of circular polarization components $I_\pm(x, y) = |\boldsymbol{\varepsilon}_\pm^* \cdot \mathbf{E}_L(x, y)|^2$, where $\boldsymbol{\varepsilon}_\pm = \frac{1}{\sqrt{2}}(\boldsymbol{\varepsilon}_x \pm i\boldsymbol{\varepsilon}_y)$, we obtain for $\phi = \pi$

$$I_\pm(x, y) = 2E^2[\cos(ky) \mp \sin(kx)]^2 \tag{6}$$

and for $\phi = 0$

$$I_\pm(x, y) = 2E^2[\cos^2(ky) + \cos^2(kx)]. \tag{7}$$

We have plotted these circular polarization components in Fig. 2. These two situations are very different.

For $\phi = \pi$, we observe alternating sites of pure σ_+ and σ_- polarization with a period equal to the wavelength. This means the light shifts of $m_F = \pm F$ Zeeman sublevels are periodic in space with opposite phases, and the same is true for the optical pumping process which always populates the ground state m_F sublevel of lowest energy. As a consequence, Sisyphus cooling [11] can take place if the laser beams are tuned correctly.

On the contrary, for $\phi = 0$ we have $I_+(x, y) \equiv I_-(x, y)$, both having a period equal to a half-wavelength. The polarization is thus linear everywhere, which precludes Sisyphus cooling. However, there is still a strong polarization gradient, as can be seen in the right graph of Fig. 2b, where we have plotted the polarization vector as a function of position in the optical lattice plane. Thus, it is probable that another sub-Doppler cooling mechanism takes place in this situation, for example, a mechanism similar to that operating in σ^+–σ^- molasses [11].

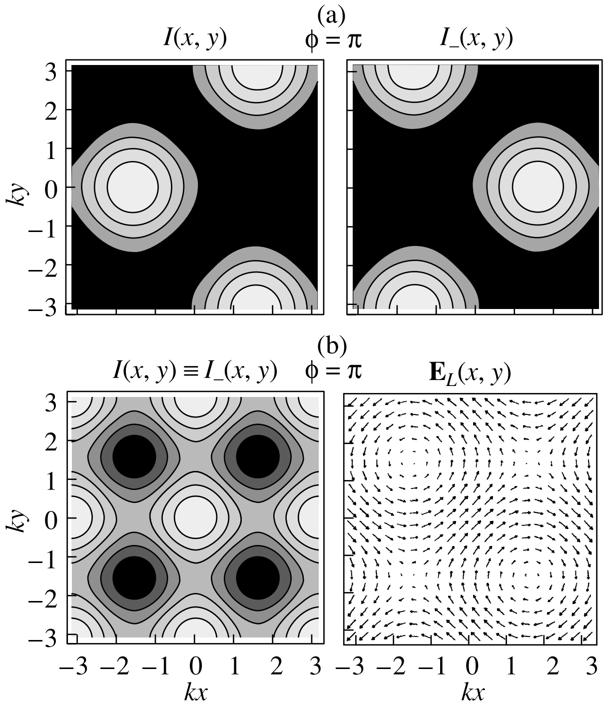

Fig. 2. Representation of the polarization gradients for the 2D optical lattice shown in Fig. 1. (a) Case where the phase of the first laser beam is $\phi = \pi$. We have plotted the intensity of circular polarization components $I_\pm(x, y)$ from Eq. (6). In this case, sites of pure σ_+ and σ_- polarization are in alternation every half-wavelength. (b) Case where the phase of the first laser beam is $\phi = 0$. On the left we have plotted $I_\pm(x, y)$ from Eq. (7). In this case $I_+(x, y) \equiv I_-(x, y)$, therefore the polarization is linear everywhere. However, there is still a strong polarization gradient, as can be seen on the right graph where we have plotted the polarization vector field $\mathbf{E}_L(x, y)$ in the lattice plane.

We conclude that the instability of the optical lattice leads to dramatic changes in the polarization gradients. This is a crucial problem when one works with cooling mechanisms involving the spatial dependence of light polarization. One solution to this problem is to stabilize mechanically the phase difference between laser beams, as was first implemented by Hemmerich *et al.* [12]. However, other approaches free of this mechanical constraint were proposed by the groups of Grynberg [8] and Meschede [9]. We shall discuss both of these approaches in Sections 4 and 5, but first we start by defining intrinsic phase stability and by establishing a necessary and sufficient condition for it.

3. NECESSARY AND SUFFICIENT CONDITION FOR INTRINSIC PHASE STABILITY

Let us consider an optical lattice composed of l laser beams, and let us suppose that the phases of these laser beams change suddenly as follows:

$$\phi_j \longrightarrow \phi_j + \Delta\phi_j. \quad (8)$$

From Eq. (3), we see that it is possible to compensate the effect of this change by a translation $\mathbf{r} \longrightarrow \mathbf{r} - \mathbf{\Delta r}$, provided there exists a vector $\mathbf{\Delta r}$ and an arbitrary phase ϕ_o (which has no effect on $\hat{U}(\mathbf{r})$) such that

$$\mathbf{k}_j \cdot \mathbf{\Delta r} = \Delta\phi_j + \phi_o, \quad \forall j = 1, \ldots, l. \quad (9)$$

This brings us to the following definition: we say that an optical lattice is *intrinsically phase-stable* if any change in the phases of the laser beams can be compensated by a formal translation as explained above. In practice, this means that every change in the phases of the laser beams manifests itself as a physical translation of the optical lattice in space. Such translations do not disturb the optical cooling mechanisms as long as the atoms' internal variables evolve much more rapidly than the optical potential, which is usually the case.

Let us now state an important result. We denote by $\mathcal{K}$ the matrix composed of the lines $(\mathbf{k}_j - \mathbf{k}_1)^T$ for $j = 2, \ldots, l$, and $\mathbf{\Phi}$ is the vector composed of the elements $\Delta\phi_j - \Delta\phi_1$ for $j = 2, \ldots, l$. We recall that the rank of a matrix is equal to the dimension of the vector space generated by its columns or by its rows. Then, the optical lattice resulting from the superposition of the l laser beams is intrinsically phase-stable if and only if

$$\mathrm{rank}(\mathcal{K}) = \mathrm{rank}(\mathcal{K}|\mathbf{\Phi}), \quad \forall\mathbf{\Phi} \quad (10)$$

where $(\mathcal{K}|\mathbf{\Phi})$ is the matrix obtained from $\mathcal{K}$ by adding a column composed of the elements of $\mathbf{\Phi}$.

The proof of this result is as follows. It is clear that the optical lattice is intrinsically phase-stable if and only if the linear system (9) always admits a solution for any choice of the phase variations $\Delta\phi_j$. By subtraction of the first equation, we obtain the following linear system:

$$\{(\mathbf{k}_j - \mathbf{k}_1) \cdot \mathbf{\Delta r} = \Delta\phi_j - \Delta\phi_1 | j = 2, \ldots, l\}, \quad (11)$$

which is equivalent to system (9). System (11) can be written in matrix form as $\mathcal{K}\mathbf{\Delta r} = \mathbf{\Phi}$. For this equation to have at least one solution, it is both necessary and sufficient that $\mathrm{rank}(\mathcal{K})$ be equal to $\mathrm{rank}(\mathcal{K}|\mathbf{\Phi})$ (this is the Rouché–Capelli theorem for the existence of the solution of a linear system). Here ends the proof.

4. MINIMAL OPTICAL LATTICES

Let us consider an optical lattice composed of l laser beams, and let us denote by d its spatial dimension. We will demonstrate that $d = l - 1$ is a sufficient condition to guarantee the intrinsic phase stability of the optical lattice. Let us start by a demonstration of the following properties:

1. $d = \mathrm{rank}(\mathcal{K})$;
2. $\mathrm{rank}(\mathcal{K}) \le \mathrm{rank}(\mathcal{K}|\mathbf{\Phi})$;
3. $\mathrm{rank}(\mathcal{K}|\mathbf{\Phi}) \le l - 1$.

As can be seen from Eq. (3), the optical lattice is generated by the vectors $(\mathbf{k}_j - \mathbf{k}_i)$; thus, its spatial dimension d is equal to the dimension of the vector space generated by the vectors $(\mathbf{k}_j - \mathbf{k}_i)$. But this vector space is also generated by the vectors $(\mathbf{k}_j - \mathbf{k}_1)$ which compose the matrix $\mathcal{K}$. Therefore, by definition of the rank, d is equal to $\mathrm{rank}(\mathcal{K})$, which proves the first property. The second property comes from the trivial assertion that adding a column to a matrix cannot decrease the rank. Finally, the rank of a matrix cannot exceed the number of lines, and this proves the last inequality.

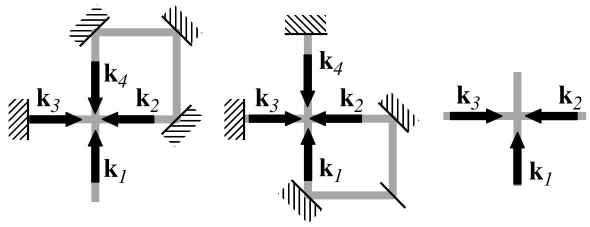

Fig. 3. Comparison of optical lattices of dimension $d = 2$: (a) folded optical lattice; (b) unstable optical lattice; (c) minimal optical lattice (with $d = l - 1$).

Writing these three properties side by side, we get

$$d = \mathrm{rank}(\mathcal{K}) \le \mathrm{rank}(\mathcal{K}|\Phi) \le l - 1. \quad (12)$$

From this expression, it is obvious that by imposing the condition $d = l - 1$ we guarantee that $\mathrm{rank}(\mathcal{K}) = \mathrm{rank}(\mathcal{K}|\mathbf{\Phi})$ and, therefore, that the optical lattice is intrinsically phase-stable. This is the solution proposed by Grynberg *et al.* in 1993 to build stable optical lattices [8]. Note that Eq. (12) implies $l \ge d + 1$. Therefore, $l = d + 1$ is the *minimum* number of laser beams needed to create an optical lattice of dimension d, hence the term *minimal optical lattice.*

At this point, it is important to note that the condition $d = l - 1$ is sufficient, but not necessary, to have $\mathrm{rank}(\mathcal{K}) = \mathrm{rank}(\mathcal{K}|\mathbf{\Phi})$. There is another method to obtain an optical lattice which is intrinsically phase-stable. It is described in the next section.

5. FOLDED OPTICAL LATTICES

Let us consider the 2D optical lattice geometry presented in Fig. 3a. This optical lattice is intrinsically phase-stable even though it does not satisfy the condition $d = l - 1$. To explain this point, we start by considering the optical lattice of Fig. 3c, which is composed of the first three laser beams $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$. This optical

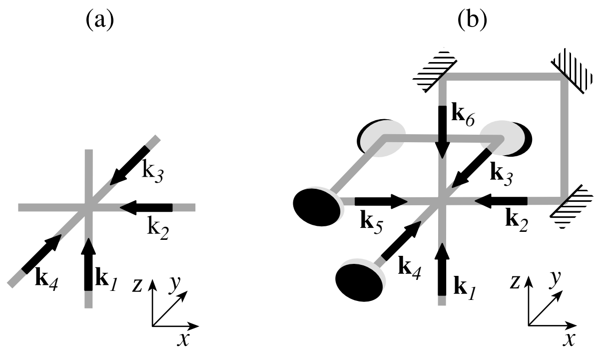

Fig. 4. Three-dimensional optical lattices: (a) minimal (with $d = l - 1$); (b) folded.

lattice is intrinsically phase-stable, since it satisfies the condition $d = l - 1$. Therefore, we have

$$\mathrm{rank}\begin{pmatrix} \mathbf{k}_2^T - \mathbf{k}_1^T \\ \mathbf{k}_3^T - \mathbf{k}_1^T \end{pmatrix} = \mathrm{rank}\left(\begin{array}{c|c} \mathbf{k}_2^T - \mathbf{k}_1^T & \Delta\phi_2 - \Delta\phi_1 \\ \mathbf{k}_3^T - \mathbf{k}_1^T & \Delta\phi_3 - \Delta\phi_1 \end{array}\right) = 2. \quad (13)$$

Let us now reconsider the optical lattice of Fig. 3a. Since the retro-reflected beam follows the same path as the incident beam, the phases satisfy the relation $\phi_4 - \phi_3 = \phi_2 - \phi_1$. Therefore, the differences $\phi_j - \phi_1$ are linked via

$$(\phi_4 - \phi_1) = (\phi_3 - \phi_1) + (\phi_2 - \phi_1). \quad (14)$$

On the other hand, the differences $\mathbf{k}_j - \mathbf{k}_1$ are related by

$$(\mathbf{k}_4 - \mathbf{k}_1) = (\mathbf{k}_3 - \mathbf{k}_1) + (\mathbf{k}_2 - \mathbf{k}_1). \quad (15)$$

Since the linear combinations (14) and (15) are identical, we have

$$\mathrm{rank}\left(\begin{array}{c|c} \mathbf{k}_2^T - \mathbf{k}_1^T & \Delta\phi_2 - \Delta\phi_1 \\ \mathbf{k}_3^T - \mathbf{k}_1^T & \Delta\phi_3 - \Delta\phi_1 \\ \mathbf{k}_4^T - \mathbf{k}_1^T & \Delta\phi_4 - \Delta\phi_1 \end{array}\right) = \mathrm{rank}\left(\begin{array}{c|c} \mathbf{k}_2^T - \mathbf{k}_1^T & \Delta\phi_2 - \Delta\phi_1 \\ \mathbf{k}_3^T - \mathbf{k}_1^T & \Delta\phi_3 - \Delta\phi_1 \end{array}\right) \quad (16)$$

and thus $\mathrm{rank}(\mathcal{K}|\mathbf{\Phi}) = 2$. Now, using Eq. (12) with $d = 2$, we can conclude that $\mathrm{rank}(\mathcal{K}) = \mathrm{rank}(\mathcal{K}|\mathbf{\Phi})$ is always satisfied, and therefore the optical lattice of Fig. 3a is intrinsically phase-stable.

The idea of using this type of intrinsically phase-stable configuration was initially put forward by Rauschenbeutel *et al.* in a slightly different form [9]. Their explanation for stability is more intuitive and consists in observing that the optical lattice of Fig. 3a is created by folding a 1D lattice such that it intersects with itself. Since 1D lattices are intrinsically stable, as discussed above, folded ones must be too.

One can also say that the stability is preserved when we add a fourth laser beam, because the phase and wave vector of this laser beam are related to the phases and wave vectors of the other laser beams by the same linear combination, as shown by Eqs. (14) and (15). Indeed, if we consider the configuration of Fig. 3b, relation (15) is still satisfied but relation (14) is not, since the phases are all independent. Therefore, we have $\mathrm{rank}(\mathcal{K}) = 2 < \mathrm{rank}(\mathcal{K}|\mathbf{\Phi}) = 3$ and the optical lattice is not intrinsically phase-stable.

6. DISCUSSION

6.1. Optical Lattices in 1D and 3D

Although all the examples given above were 2D lattices, everything we have said is still true in other dimensions. For dimension $d = 1$, the minimal and folded optical lattices are degenerate and correspond to the usual optical molasses. In this case, the intrinsic phase stability is obvious even without the above matrix analysis, because any displacement of the retro-reflecting mirror automatically shifts the phase of the standing wave by a corresponding amount, since the electric field has a node on the mirror surface. For dimension $d = 3$, the minimal and folded optical lattices are generalizations of the two-dimensional case. An example of a minimal optical lattice in three dimensions is presented in Fig. 4a. This configuration has been used by Treutlein *et al.* for degenerate Raman sideband cooling [13]. Although the geometry was not symmetrical, radiation pressure was reduced by using a large detuning. Other examples of minimal optical lattices are discussed in [14]. An example of a three-dimensional folded optical lattice is presented in Fig. 4b. This configuration is obtained from configuration (a) by adding two laser beams, namely $\mathbf{k}_5$ and $\mathbf{k}_6$. It is easy to show that the phases and wave vectors of these two beams are related to the other beams by the same linear relations:

$$\phi_5 - \phi_1 = (\phi_4 - \phi_1) + (\phi_3 - \phi_1) - (\phi_2 - \phi_1), \quad (17)$$

$$\phi_6 - \phi_1 = (\phi_4 - \phi_1) + (\phi_3 - \phi_1), \quad (18)$$

and

$$\mathbf{k}_5 - \mathbf{k}_1 = (\mathbf{k}_4 - \mathbf{k}_1) + (\mathbf{k}_3 - \mathbf{k}_1) - (\mathbf{k}_2 - \mathbf{k}_1), \quad (19)$$

$$\mathbf{k}_6 - \mathbf{k}_1 = (\mathbf{k}_4 - \mathbf{k}_1) + (\mathbf{k}_3 - \mathbf{k}_1). \quad (20)$$

Therefore, the optical lattice is intrinsically phase-stable.

6.2. Practical Realizations of Optical Lattices

Work on cold atoms usually requires one to employ a symmetrical beam configuration in order to avoid atoms being pushed aside by radiation pressure. Minimal optical lattices can be designed in a symmetrical geometry, but this requires a complex vacuum system. For the 2D case, this means using 3 beams intersecting at 120° and a hexagonal coplanar geometry for the vacuum system. To create a symmetrical 3D minimal lattice, the 4 beams should form a regular tetrahedron. This adds even further to the complexity of the vacuum apparatus (see [15] for an example of a tetrahedral magneto-optical trap).

Folded lattices, on the other hand, involve more beams but have a more user-friendly geometry with beams intersecting at right-angles. They can be aligned by auto-collimation and they have the inherent advantage of balanced radiation pressure. In addition, it is straightforward to adapt them to a power recycling geometry, a tremendous advantage in many cases.

6.3. Atomic Beam Collimation with a Folded Lattice

In a recent experiment, we used the 2D folded optical lattice of Fig. 3a to perform the collimation of a continuous cesium beam in a fountain geometry [16]. In this folded lattice, we realized Zeeman shift degenerate-Raman-sideband cooling in a continuous mode. This powerful cooling technique allowed us to reduce the atomic beam transverse temperature from 60 μK to 1.6 μK in a few milliseconds. We note that, in this context, power recycling is highly advantageous, since a high power is necessary to create a far-off-resonance optical lattice.

With the same experimental setup, we also realized collimation of the continuous cesium beam using Sisyphus-like cooling in a 2D optical lattice. We have experimented with the lattice configurations (a) and (b) of Fig. 3 and the results are summarized in the table. The best collimation was obtained with the folded optical lattice.

6.4. Multicolor Optical Lattices

Before concluding, we should like to point out that fulfilling condition (10) is by no means the only solution of the instability problem. Another possibility is to average over the phase difference. To illustrate this, consider the unstable 2D optical lattice of Fig. 1. If the two molasses have different laser frequencies, the phase difference changes rapidly, and the atoms see an optical lattice which is the average of the optical shift over the phase variable. This solution has been used with success by other groups [17, 18].

Summary of transverse temperatures obtained in the collimation experiment with Sisyphus-like cooling in lattice configurations (a) and (b) of Fig. 3. See Subsection 6.3 for details

Optical lattice configuration	Transverse temperature, μK
(a) Folded	3.6(2)
(b) Unstable	7.3(5)

7. CONCLUSIONS

In this article, we have established a necessary and sufficient condition for the intrinsic phase stability of an optical lattice. We have presented two practical solutions to fulfill this condition, namely, minimal and folded optical lattices. We have shown that the minimal optical lattices, introduced for the first time by Grynberg *et al.* in 1993, are sufficient but not necessary for stability. Indeed, another possibility is to use a folded optical lattice, as proposed by Rauschenbeutel *et al.* in 1998. We have presented a particular example of a folded optical lattice, which has the advantages of power recycling, symmetry, and a more convenient geometry. Henceforth, such a lattice would seem to be a more natural choice for most experiments. Indeed, for many applications a folded lattice looks like a better source of cold atoms than a conventional six-beam optical molasses.

ACKNOWLEDGMENTS

This work was supported by the Swiss National Science Foundation, the Swiss Federal Office of Metrology and Accreditation (METAS), the canton of Neuchâtel, and the Swiss Confederation. AVT and VIYu were supported in part by INTAS, grant no. INTAS-01-0855, and by the Russian Foundation for Basic Research, grant no. 04-02-16488.

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